Discrete Unit
Probability

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Lesson Plans
Counting Principle and Combinations

Middle Level

Prior Knowledge:
Students should be able to

- represent numbers in various ways
- create and solve word problems using actions, objects, words, and pictures
- demonstrate mastery of the concept of multiplication
- collect and record data

Curriculum Integration:
Standards correlated to the content in this unit are

- Mathematics as Problem Solving
- Mathematics as Communication
- Mathematics as Reasoning
- Number and Number Relationships
- Patterns and Functions

Objective:
Students will

- use pictures to represent a real-life problem
- build a tree diagram to represent real-life problems
- find patterns in relationships to develop rules
- correctly use the counting principle for simple real-world problems

Day 1: **Trophy Troubles**

Activity from Hands on Math for Middle Grades, by Robert Smith, © 1996, Creative Teaching Press, Inc., pages 32-33, 80.

Objective: Introduce the concept and notation of factorial

Students will need scissors, calculator, and copies of page 80.

ACTIVITY:

A. Have students cut out the first two trophies, Most Considerate and Best Athlete, from page 80.

B. As a class discuss and record all the different ways you can order these two trophies. There are only two ways: either MC is first with BA following or BA is first with MC following.

C. Have students begin the following work with partners. Cut of the third trophy for Super Speller. Arrange the three trophies (first, second, and third) on your desk as many different ways as you can. Have students use abbreviations to record the different possible arrangements.

D. Have students cut out a fourth trophy and arrange the four in as many different ways as they can. Students should record the arrangements.

E. As a class, ask students to speculate how many arrangements could be used for five trophies.

Help students to examine the pattern:

2 trophies $2 * 1 = 2$

3 trophies $3 * 2 * 1 = 6$

4 trophies $4 * 3 * 2 * 1 = 24$

5 trophies ???

Problems like these are factorials. Use the factorial pattern to calculate how many different ways 5, 6, or 7 trophies can be arranged.

F. Extension questions. Would you get the same answers if you were arranging four different baseball cards? Give reasons. You and your siblings are, believe it or not, arguing about who should sit where at the dinner table. Draw diagrams illustrating all the possible arrangements. List three other practical ways to use factorials in everyday living.

Day 2: **Ice-cream Cones**

Objective: Have students explore the counting principle by representing the following problem with pictures

Students will need a copy of the handout "ICE-CREAM CONES" found at the following website: http://illuminations.nctm.org/lessonplans/6-8/combinations-68/CONE.JPG

** Suggested adaptation: Have order of ice cream on cone matter.
First scoop chocolate second vanilla is different than first vanilla, second chocolate.

Students will also need eight different colored pencils

ACTIVITY:

A. Begin by exploring this problem with the whole class.

* Suppose you go to your friends house and are offered a double-dipped cone. (Two different scoops of ice-cream) How many different double-dip cones can be made from by choosing either vanilla, chocolate or strawberry ice-cream. Have students record all the possible outcomes.

B. Once the class understands this problem, give students the worksheet.
Allow them to work in groups of two to four.

C. As a class, conclude by discussing, sharing and comparing group's answers
Focus discussion on how students know they are not missing any combinations

D. Extensions: Is there any relationship between these answers and using factorial?

Assessment: Informal throughout group work time and class discussion time

Day 3: **Tree Diagram**

Objective: To solve the ice-cream cone problem using a tree diagram

ACTIVITY:

A. As a class, begin looking at other ways to organize and list possibilities for a double-dipped cone with only three choices of ice-cream. Have student share ideas to come up with different ways to list.

Draw a tree diagram. Explaining what each branch represents and how to know when to stop. List all the outcomes from the tree diagram. Ask students to compare this work with yesterday's work.

B. Pose similar, more complex questions for groups of three to four students to solve. Have students work on organizing a way to list all possibilities. Have students work of making tree diagrams.

How many different two-scoop cones can be made from six different ice-cream choices?

How many different three-scoop cones can be made from five different ice-cream choices?

How many different two-scoop cones can be made from five different ice-cream choices?

How many different three-scoop cones can be made from six different ice-cream choices?

C. As a class, discuss relationships that students observe from working with these problems.

Emphasize ideas that will lead to thinking about the counting principle

D. Extensions: Can you think of another problem you could answer by making a tree diagram?

Assessment: Ask students to speculate using some of the class generalizations some similar problems.

E. Extra practice assessment: As homework have students work on the following worksheet:
http://illuminations.nctm.org/lessonplans/6-8/combinations-68/circle.htm
This activity can be used to allow students added opportunity to pictorially explain combinations and use rules to explain how many ways the circles can be colored.

Day 4: **Crayon Combos**

Activity from Hands on Math for Middle Grades, by Robert Smith, © 1996, Creative Teaching Press, Inc., pages 68-69

Objective: Students will begin to develop an understanding of combinations as well as explore patterns

Students will need crayons

ACTIVITY:

A. Review the ice-cream cone activity. Establish that this activity will not be exactly the same. Have students select these four colors: red, green, blue, and yellow. As a class determine how many different color combinations you can make using these four colors. Remind the students that red/green is the same combination as green/red. Have students record all the different combinations in a chart.

B. Have students work with partners to explore how many different combinations can be made from five colors, six colors, seven, eight colors. Have students record all their combinations in a chart.

C. Have students who have a clear organizational pattern record their combinations on the board. As a class discuss the patterns that are noticeable. What number patterns could you find for the combinations as you went from the first to the last set? Why was using eight colors much more complicated than using four colors? How many combinations can be made using nine or ten colors? What is the rule to decide how many two-color combinations there would be for any number of crayons?

D. Have students create a tree diagram for using five colors to help reinforce the pattern.

E. Extensions: Why did the number of color combination possibilities decrease each time you started with a new color? You have eight different schoolbooks at home. You can only bring three books to school on any day. How many different combinations of books can you carry to school?

Day 5: **Let Me Shake Your Hand**

Objective: Have students develop a understanding of a variation of the counting principle

Students will need a copy of the handout "Let Me Shake Your Hand"

ACTIVITY:

A. Have class break into groups of 3.
   Have each student shake the hand of everyone else in the group.
   Have students record the total number of hand shakes that happened in their group.

B. Have students break up into groups of 4.
   Have each student shake the hand of everyone else in the group.
   Have students record the total number of hand shakes that happened in their group.

C. Have students now break into groups of 5.
   Have each shake everyone else's hand.
   Have students record the total number of hand shakes that happened in their group.

D. Ask students to develop a table for recording this data.
   Have students share their results with their group of 5.
   Make sure all students in the groups agree on the data that is in everyone else's table.

E. Ask students in these groups to speculate the number of handshakes that would take place for everyone in the whole class to shake everyone else's hand.
   Have students provide support for their answer.

F. As a whole class, discuss what the students have noticed include the counting principle.
   Have students record class observations on their worksheet.

G. As a class categorize all the problems that have been done as being either problems when order matters or doesn't matter. Emphasize the difference mathematically between the two. This discussion should serve as a connection between each of the problems and can be used as an informal assessment of were the students are in their learning.

H. Extension questions: Can you make a rule to describe this relationship? Why must each of those steps be a part of the rule according to the situation? Can you think of any other real world problems that would have relationships like this one?

Assessment: Check explanations of parts E&F on the worksheet.

Day 6/7: **Food Combinations**
Final Unit Project

Objective: Allow students to formalize and display their understanding of combinations and tree diagrams.

ACTIVITY:

A. Have students work individually as a final assessment. Give students the situation of wanting to develop a menu at their new restaurants. Customers want to be able to have at least 100 choices of meals to eat. Students will need to choose at least three menu categories and list all the items in that category. For example: Soups might be a category and tomato, chili and cheese would be three soups in that category. Other categories might be main dish, beverage, desserts. Have students use a tree diagram to organize all the different combinations. Have them write out a rule to justify the number of combinations they counted.

B. Have students design either a smaller scale of the menu they would hang in the restaurant or give to customers as they walk in. Have the students come up with an advertising claim about how many combinations can be made at their restaurant. Ask students how the Fundamental Counting Principle could be used to come up with this number. Ask students to describe why this works.

C. Extensions: Students could be asked to come up with exactly 300 combinations. Students could also be asked when the situation might arise where they need to eliminate any choices because of the same combination coming up twice.

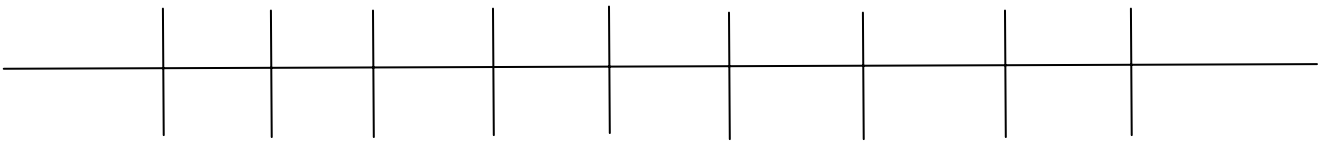
Let Me Shake Your Hand

A. How many handshakes total were there in your group of three people?

B. How many handshakes total were there in your group of four people?

C. How many handshakes total were there in your group of five people?

D. Use this data to begin to complete the table below.



E. With the group you are in now, predict how many handshakes would take place if the group was six people. Describe how you arrived at your answer.

F. With your group, predict how many handshakes would take place if the group 25 people. Describe how you arrived at your answer.

G. List other observations from the class discussion.

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Overview

Content Standard: Probability ***Level:*** Partial (7^{th}/8^{th} grade)

Specific Statement(s) from the Standard:
From the Minnesota Academic Standards, Mathematics K-12, May 19, 2003:
Grades 9,10,11
Probability Standard: Use appropriate counting procedures…
The student will:
1. Select and apply appropriate counting procedures to solve real-world and mathematical problems, including probability problems.

Product:
Students will design a menu at a restaurant. They will use this menu to count all the combinations of items possible; they will also display this pictorially.

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FEEDBACK CHECKLIST FOR TASK 1

The purpose of the checklist is to provide feedback to the student about his/her work relative to the content standard. Have the standard available for reference.

Y=Yes
N=Needs Improvement

Student		Teacher
_____	Student will develop sets/categories of items for the menu.	_____
_____	Student will use a tree diagram to list all combinations of items.	_____
_____	Students will use the fundamental counting principle to justify their answer.	_____

Overall Comments (information about student progress, quality of the work, next steps for teacher and student, needed adjustments in the teaching and learning processes, and problems to be addressed):