

Lesson Plans  
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Grade 5 Probability  
All reference materials come from Math Trailblazers – (M.T.)  
A TIMS Curriculum  
Unit 7

## Flipping One Coin

Day One: Students predict the number of times heads will show if they flip a coin 40 times. They then flip a coin and compare their results with their predictions. They discuss the fact that flipping a head and flipping a tail are equally likely (each has probability  $1/2$ ), and they explore other situations that have outcomes that are not equally likely.

Key Content:                   \* Collecting and analyzing data from a random process.  
                                     \* Understanding that random events are predictable “over the long run”.  
                                     \* Linking qualitative and quantitative characteristics of probabilities.  
                                     \* Exploring situations which have outcomes that are not equally likely.

Key Vocabulary:           certain  
                                     impossible  
                                     equally likely  
                                     probability

Materials: for the student:  
                                     Flipping the Coin Activity (student guide M.T.) pgs.259-261.  
                                     Three-column Data Table (Unit Resource Guide, Generic Section, M.T.)  
                                     one copy per/student  
                                     Pennies, one per student group.  
                                     Small paper cups, one per student/group  
                                     Small cloth for tossing coins (to muffle sound) per student/group.

Before this Unit:                   Students who have had this curriculum in earlier grades have had some experience with probability. Many activities in first and second grade introduce students to sampling and random variation along with basic data collection and analysis. They were introduced to probability in fourth grade.

Developing the Activity:  
Tell students you are going to flip a coin and ask them to predict whether it will come up heads or tails. Do they think one side is more likely to show than the other? Do they understand that the side that shows after the flip doesn't depend on which side is showing before the flip (as long as the coin spins around many times in the air)? Do they realize that the probability of heads showing is  $1/2$  or 50%?

Day 1 - Flip the coin and ask whether the result agrees with their prediction. Ask whether the same side would come up each time if you were to flip the coin several times. This is what they will investigate in this activity.

Ask students to follow the instructions on the Flipping One Coin Activity page 259 –261, (Math Trailblazers). In question 1. they predict how many times heads and how many times tails will show up in 40 flips. Then, in questions 2-4, they flip a coin 40 times and record their data, and compare their predictions with their results. From what they know about probability, many students will be able to predict that heads will come up about half the time, or in about 20 out of 40 flips. However, this does

not mean that heads will come up exactly half the time. However, it is true that if they flip the coin many times, the fraction of heads that appears will be close to one half.

Ask students to read the probability Discussion (M.T.pg.260). It defines probability as a number that describes how likely an event is to happen. A probability is a number from 0-1, or equivalently, a percent from 0%-100% (since 100% means one whole). Events that are impossible have a probability 0. The more likely an event is to happen, the larger the probability. Events that are certain have the largest probability.

In question 6, students decide that the probability of flipping a head is  $\frac{1}{2}$  or 50%. Ask the students to think of other events and to estimate the probability the events will occur. What are the probability students will have homework tonight? What is the probability it will snow where they live?

We say that flipping heads and flipping tails are equally likely to occur. Questions 7A-7C describe other situations with two possible outcomes: being right- or left- handed, placing right or left thumb on top when clasping hands together, having a first born boy or a first born girl. Discuss these events with your class. Your discussion will depend on your class results, but your data will probably suggest that Questions 7A-7B describe events that are not equally likely. The events in Question 7C, having a girl or boy born first are most equally likely.

### Flipping Two Coins

Days Two –Six: Students investigate probability by gathering data: a penny and a nickel are flipped repeatedly and the number of heads showing is recorded. The percentages of trials showing 0, 1, and 2 heads are calculated and graphed, then compared with the respective probabilities. Data is compared for 10, 100, and 1000 trials (after pooling data with classmates). This comparison helps students understand that probability predicts the behavior of random systems over the long run.

Key Content ..... \* Day 2 – Analyzing fair and unfair games.  
..... \* Day 3 - Listing all the ways 0,1, and 2 heads can show when 2 coins  
are ..... flipped.  
..... \* Day 3 - Computing probabilities for all possible outcomes when 2  
coins ..... are flipped.  
..... \* Day 4 - Expressing probabilities as percents.  
..... \* Day 5 - Understanding that as the number of trials increases, the results  
..... of coin flipping more closely resemble the probabilities.

Key Vocabulary ..... fair game  
..... fair  
..... unfair  
..... probability

Materials ..... For the student  
..... Flipping Two Coins Activities Pages (M.T. student guide) pgs. 262- 268.  
..... 100 Two Coin Flips activity page (M.T .Discovery Assignment book) pg.  
..... 119.  
..... Coin Flipping Data Tables Activity Page (M. T. Discovery Assignment  
..... book) pg. 121.  
..... Comparing Probability with Results Activity Page (M.T. Discovery  
..... Assignment Book) pg.123.

- .....Centimeter Graph Paper (M.T Unit Resource Guide, Generic Section)
- .....three sheets per/student.
- .....One penny per/student group.
- .....One nickel per/student group.
- .....One small paper cup for shaking coins, per/student group, optional
- .....Calculators TI73 or equivalent if possible.
- .....
- .....For the Teacher
- .....Transparency of Coin Flipping Data Tables Activity Page ( M.T.
- .....Discovery Assignment Book) pg. 121, optional.

Day Two – Ask students to play the game - How Many Heads? – in the Flipping Coins Activity Pages in the Student Guide as an informal introduction to the lab. This is a game for three players. The players flip two coins ( they can use a penny and a nickel ) and count the number of heads that show. One of the players is designated to be the 0 heads player, who will receive one point if 0 heads show. The next player is the 1 head player who will receive a point if 1 head shows, and the third player is the 2 heads player who will receive a point if 2 heads show. The questions about probability that students will investigate in the lab are introduced as the students play the game and analyze the data. Students will notice after playing the game a few times that one head shows more often (about twice as often) than either 0 heads or 2 heads. The reason for this will be more apparent after the lab. The one head player has a better chance of winning, and, for this reason, the game is inherently unfair. A **fair game** is one in which players have an equal chance of winning. Students sometimes confuse “playing fairly” (taking turns, not cheating, etc.) with the mathematical concept of a fair game. Explain that mathematicians categorize games as **fair** or **unfair** based on the probabilities have of winning rather than on the sportsmanship of the players.

You can assign Questions 1-4 in the Homework section of the Flipping Two Coins Activity Pages (M.T. student guide) at this time.

Day Three - Using Homework Question 5, Game : Matching 2 Pennies as a class activity instead of a homework assignment. Have the students break into groups of two and give them two pennies. Each student flips one penny for each turn. If the pennies match then player one gets a point, and if they are both different then player two gets a point. The first player to score 10 points wins.

- A. Have them play several games and keep track of the final scores.
- B. List the possible outcomes of flipping two coins.
- C. For which of the possible outcomes you listed in B will player 1 get a point? For which outcomes will player two score a point?
- D. Is this a **fair** game (do each of the players have an equal chance of winning)?  
Explain your answer(have each student do this individually as an assessment)

Day 4 - Lab : Divide the class into 10 groups. Each group of students will flip two coins, a penny and a nickel, 100 times. Then, they will combine their data with the other groups’ data to get the results of 1000 flips. If you have fewer than ten groups in your class, you can have volunteers carry out the additional sets of 100 flips.

Question 4 ( Student Guide) pg.263 asks students to predict the number of times 0, 1, and 2 heads will come up in their 100 trials. Encourage students to make their predictions based on their experience with the game. In the game, 1 head showing is twice as likely as 0 heads showing and also twice as likely as

2 heads showing. So, if the two coins are flipped 100 times, 1 head should show about twice as often as either 0 heads or 2 heads. Therefore, a sophisticated prediction is fifty 1 – head flips, twenty-five 0 head flips, and twenty-five 2 head flips. Have students explain their answers individually in writing and hand them in to the teacher. Use this as an assessment.

#### Part 1. Data Tables:

After students have made their predictions, they are ready to collect their data. Question 5. They will record the outcome of the penny first and the nickel second. Students record their outcomes on the 100 Two-Coin Flips Activity Page in the (M.T.) Discovery Assignment Book. Next, they summarize the results of their first 10 flips and their 100 flips using the first two tables on the Coin Flipping Data Tables Activity Page in the (M.T.) Discovery Assignment Book. Students will pool their data from the day before with the other groups to obtain results for 1000 flips and record their pooled data in the 1000-Trial Table on the Coin Flipping Data Tables Activity Page. To help in pooling data, you may want to make a big table on the board or overhead for each group to record its 100-flip data. Then the other groups can just copy from the board or overhead.

Day 5 – Lab : Question 9 asks students to make three graphs. Ask students for different examples and list them on the board. Ask students to think about whether bar graphs or point graphs make more sense here. The line on a point graph would suggest that we can consider in-between values. For example, the line would let us predict how many times 1.5 heads show, but it doesn't make sense to talk about 1 and 1/2 heads. Since the only numbers on the horizontal axis that are meaningful are 0, 1, and 2 heads, it makes sense to use a bar graph.

Be sure students label the axes and that they put the numbers on the lines, instead of in the spaces. Students should use the same scale on all three graphs so they will be able to make comparisons among the graphs. You may want to use a transparency of Centimeter Graph Paper to model the graph for the first 10 flips.

Question 10 asks students to analyze their graphs. Students should notice that the tallest bar for every graph is the middle bar—the bar for one head. On the graph for 1000 trials, this bar is about twice as tall as the other two bars.

Day 6 – Lab : Students see in Question 11 that there are two ways one head can come up (HJT and TH), but only one way two heads can come up (HH), and one way two tails can come up (TT). This helps them understand why one head shows about twice as often as 0 and 2 heads and why the bar for 1 head is twice as tall as the other bars.

Question 14 asks students to calculate the probabilities for 0, 1, and 2 heads and to enter them in the Probabilities of Coin Flipping table on the Comparing Probability with Results Activity Page (M.T.) in the Discovery Assignment Book. They record the data for 10, 100, and 1000 trials in the Results of Coin Flipping table on the same page. They obtain this data from the last column of their 10-Trial, 100-Trial, and 1000-Trial Tables. Notice that with more flips the data are closer to the probabilities.

Remind students that probability is a way of predicting what will happen over the long run.

To grade the lab you can assign points to each section.

### **Families with Two Children**

Day 7- This activity gives a real world application of the coin-flipping model. Students investigate the various number of boys a family with two children might have and compute the probability of each number of boys. Then, they make a list of families they know with two children, combine their lists to

make a class list of 100 two children families, and compare the number of families from this list that have 0, 1, and 2 boys with the number they have predicted using probability.

Key Content:                   \* Computing probabilities of having 0, 1, and 2 boys in a 2-child family.  
                                  \* Expressing probabilities as fractions and percents.  
                                  \* Using probability to predict.  
                                  \* Comparing probability with real world data.

Materials:                   For the student:  
                                  Families with Two Children Activities Pages (M.T. Student Guide) Pages  
                                  269-271

Day 7- Question 1 on the Families with Two Children Activity Pages in the Student Guide asks students to make a list of all the two-child families that are possible. They complete a data table. They use the table to answer Questions 2-3. The table should show that 1 type of family has 0 boys(GG), and 2 types of families have 1 boy (GB,BG), and 1 type of family has 2 boys(BB). Question 4 asks students to give the probability of having each type of family (if a family has two children). Since there are four possible ways to have a two-child family and one way to have no boys (GG), then the probability of having no boys is 1 out of 4 (1/4) or 25%. Using the same reasoning, there 2 out of 4 ways to have one boy (BG,GB), so the probability of having one boy is 2/4 or 50%. The probability of having two boys (BB) is 1/4 or 25%. Question 5 asks students to predict the number of families of each type they would expect in 100 families. Using the probabilities from question 4, it is reasonable to predict that about 1/4 or 25 of the families will have no boys, 1/2 or 50 will have one boy, and 1/4/or 25 of the families will have 2 boys. To complete Questions 6-7, the class compiles a list of families which have two children. Students can work in groups to list the families they know and record the number of boys in each family. Then the groups can pool the data for 100 families, tally the number of families with 0, 1, and 2 boys. To complete question 8, students compare their data from Question 7 to their predictions in Question 5. It is highly likely that the results will closely match the predictions discussed in Question 4. Students should not expect exactly 25 two boy families, 50 one boy families, or 25 no boy families.

### **Unlikely Heroes**

Day 8 - Unlikely Heroes is a story about John Kerrich and Eric Christensen and the probability experiments they carried out while in prison during World War II. Kerrich and Christensen explored what happens when a coin is flipped many times. They recorded their explorations in data tables and graphed the results of the experiments in a book.

Key Content:                   \* Exploring the probabilities involved in flipping a coin.  
                                  \* Discussing fair and unfair coin tosses.  
                                  \* Relating mathematics to the real world.

Key Vocabulary            fair  
                                  unfair

Materials                   For the Student  
                                  *Unlikely Heroes* (Adventure Book) Pgs. 35-46.  
                                  Pennies optional

Day 8- Students will read the story. When the students are finished reading ask the students these discussion questions:

What do you think happens when a coin is flipped many times?  
What happened when you flipped one coin?  
Why might Christensen have said that the coin flipping experiment sounds boring?  
Why might Kerrich have gotten 4 heads and 6 tails when flipping the coin ten times?  
Do you think Kerrich is flipping a fair coin?  
Look at the data table. What do you notice about the percent of heads and tails flipped as the number of flips gets greater?  
Tell a story for the graph.

### Day 9 – 10 Game- Three-Coin Flip

Key Content:

- \* Analyzing **Fair** and **Unfair** games
- \* Listing all the ways 0, 1, and 2 heads can show when two coins are flipped.
- \* Computing probabilities for all possible outcomes when two coins are flipped.
- \* Expressing probabilities as percents.

Key Vocabulary:

- Fair
- Unfair
- Probability

Materials: Each group of students will need three coins, paper (divided into 3 columns) to record data, graph paper and a calculator.

Day 9 - Divide the students into groups of two. Give each group three coins. Tell them that they have to choose either the coins flip up all the same or flip up with some different. Each flip is worth one point. After the students have chosen, they may begin to flip the coins and record the data. The students will then begin to take turns, flipping the coins for a total of 10 flips for each student. They will record the data. Then the teacher will ask each group for their results and list the data on the board for the students to copy for their records.

Day 10 - The students will use their data from the Three-Coin Flip and as individuals, they will pick a graphing method and graph the class data. They will compute the probabilities for each possibility. They will answer the question “Was this a Fair or Unfair game”? in addition, explain why they think it is fair or unfair.

Content Standard: Probability\_\_\_\_\_

Level: Grade 5

***Specific Statement(s) from the Standard:***

NCTM:

Predict the probability of outcomes of simple experiments and test the predictions.

***Product(s):***

The student will write a paragraph telling whether a game is fair or unfair .

***Task Description:***

Students will play a game, collect data, organize data, graph data, and write a paragraph, “Is this a fair or unfair game”?

Students will explain why they made their choice.

Materials: Each group of students will need three coins, paper (divided into 3 columns), to record data, graph paper and a calculator.

Divide the students into groups of two. Give each group three pennies. Tell them that they have to choose either the coins flip up all the same or flip up with some different. Each flip is worth one point. After the students have chosen, they may begin to flip the coins and record the data.

The students will then begin to take turns, flipping the coins for a total of 10 flips for each student. They will record the data. Then the teacher will ask each group for their results and list the data on the board for the students to copy for their records.

The students will use their data from the Three-Coin Flip and as individuals, they will pick a graphing method and graph the class data. They will compute the probabilities for each possibility. They will answer the question “Was this a Fair or Unfair game”? in addition, explain why they think it is fair or unfair.

***Special Notes:***

The students are gathering data as groups and the teacher needs to write the information on the board in an unorganized way so the students have to organize the data individually.

PERFORMANCE PACKAGE TASK 1  
(Title of Package)

**FEEDBACK CHECKLIST FOR TASK 1**

The purpose of the checklist is to provide feedback to the student about his/her work relative to the content standard. Have the standard available for reference.

Y=Yes

N=Needs Improvement

<u>Student</u>		<u>Teacher</u>
_____	Data sheet is completed correctly	_____
_____	Graph is labeled correctly and done neatly	_____
_____	Conclusion is based on evidence in reference to graph	_____
_____		_____
_____		_____
_____		_____

**Overall Comments** (information about student progress, quality of the work, next steps for teacher and student, needed adjustments in the teaching and learning processes, and problems to be addressed):