Probability Unit Plans

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Lesson Plan Overview

Lesson 1: Introduce probability and explore “fair” and “unfair games”.

Lesson 2: Find empirical (experimental) data and theoretical data.

Lesson 3: Explore empirical and theoretical data.

Lesson 4: Sample space

Lesson 5: Find expected value of various games

Lesson 6: Explore expected value.

Assessment of Lessons 1-6

Lesson 7: Analyze Games and Outcomes using Area Models and other Sample Spaces

- Mazes for hiding treasures
- Using Shooting Percents to find Probability of one-on-one situations
- Best Arrangement of Marbles in a can

Lesson 8: Analyze the Sequence of Events

- Possible Outcomes of at least 3 children
- Possible Outcomes of 4 puppies
- Guessing Answers on a 4 point Quiz

Assessment of Lessons 7-8

Lesson 1**:** Introduce Probability and explore "fair" and "unfair" games

Materials: post-it notes

Steps:

1. Draw the probability number line on the board

    0----------0.5----------1.0

    Impossible Equally Likely Certain

2. On post-it notes, write 6-10 different situations with various outcomes. For example, you flip coin and get heads (0.5); you roll a six sided die, the probability of rolling a 7 (0). Place the post-it note under the appropriate heading. Make sure to also include scenarios that will fall under other categories and try to assign a numerical value to it (i.e. what is the chance of rain today, the chance of rolling a 4 on a die, etc.). Other examples can be found on the web at www.mathgoodies.com/lessons/
3. Have the students come up with other scenarios that could also go on the number line and try to assign numerical values to them as well.

Explore "fair" and "unfair" games.

Activity is from the Investigations (STEM) series, book #2.

1. Play paper, rocks, scissors in partners. The person who gets the best of 7 games wins.
2. Discuss if this is a fair or unfair game. What makes it a fair or unfair game?
3. In groups of three play again with the following conditions.

    Player A scores if all 3 signs are the same

    Player B scores if all 3 signs are different

    Player C scores if 2 signs are the same and 1 is different.
4. The first person to win 3 games wins the round.
5. Record the winners on the board along with the letter they were.
6. As a class, discuss which of the two games they just played were fair and which was unfair. What made the second game unfair?
7. Discuss other fair and unfair games that the kids have played.

Can also use the Hey, That's Not Fair! Or is it? Handout from Explorations Series Activity #9

Lesson 2: Empirical Data and Theoretical Data

Materials: Lab sheet 1.2 from CMP book What Do You Expect? (1 per group)

Spinners or paper clips to use as spinners

Steps:

1. In partners, students play matching colors 24 times (12 turns for each person). Record each turn on the lab sheet.
2. Player A scores 1 point for a match and player 2 scores 2 points if there is not a match after 2 spins.
3. Each group records the results on the board.
4. When all groups are finished explain that the data that is collected on the board is the experimental data or Empirical data. The empirical data is information you collect from experiments, playing games, etc.
5. To find empirical data of a match we can look at it as P (match) = # turns that are matches/ TOTAL # of turns. We read this as "the probability of a match equals the number of turns that are actually matches over the total number of turns that were taken.
6. Have group find their probability of a match. Assist as needed.
7. Then talk about the P (no-match) = # of turns that are not matches/ total number of turns taken.
8. Have groups find their probability of a no-match. Assist as needed.
9. Find the empirical data of the class for matches and no-matches.
10. Point out that the P (match) and P (no-match) when added together should be equal to 1. They are the only 2 possible outcomes that could take place, so we can check our work by adding them together.
11. As a class, list all of the possible outcomes that can take place in the match-no-match game. (blue, blue) (blue, yellow) (yellow, blue) (yellow, yellow)
12. Explain that in order to find the theoretical data we need to know all of the possible outcomes that **could** take place. After you identify all possible outcomes we can look for the theoretical probabilities. We find the theoretical probability the same way we found the empirical probability, but we use the outcomes that could take place from the list you just created.

    P (match) = # outcomes that are matches/ # of possible outcomes

    P (no-match) = # outcomes that are no-matches/ # of possible outcomes
13. As a class find the theoretical probabilities of the match/no-match game.
14. Discuss if the game is fair. Students should see that there are same number matches as no-matches, so the game is not fair. Brainstorm as a class how we could make the game fair.

Lesson 3: Explore empirical and theoretical data.

Materials: Lab sheet 1.3 from CMP book What Do You Expect? (1 per group)

Spinners or paper clips to use as spinners

Graph paper

Steps:

1. Students play Making Purple 50 times and record the data
2. Record the number of purples out of 50 on the board.
3. Each group should find their empirical data for making purple.
4. Students graph the P (making purple) after 5, 10, 15 …50 turns on graph paper. Make sure to include labels on the axis.
5. In groups students should also list all of the possible outcomes of making purple.
6. Find the theoretical probability of making purple.
7. When all groups are done bring them back together as a class and ask how many total outcomes could occur in this game. What is the probability of making purple? Then look at their graphs and compare the theoretical data to what they have on their graphs. Students should notice that the more times you play the game the closer they get to 0.11 (0.11 is the theoretical probability of getting purple).
8. Find the empirical data of the entire class. How close did the class's empirical data get to the theoretical data?
9. Explain that the law of large numbers demonstrates that the empirical data should get closer to the theoretical data.

You may also include the Hare and Tortoise Game, PIG, SKUNK, and Roller Derby for additional games of probability.

Lesson 4: Sample Space

Materials:

Steps:

Lists:

1. Explain that sample spaces include the lists we made for the match/no-match game and the making purple game. Lists are just 1 way we can represent all of the possible outcomes for theoretical probability.

Tables:

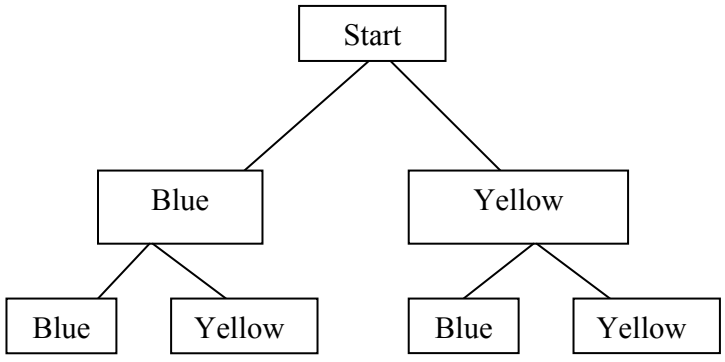
2. Similar to punnett squares in science
3. Multiply going across to list all of the possible outcomes

Ex. From match/ no-match game

*	Blue	Yellow ← (Spinner 1 options)
Blue	Blue, Blue, Blue, Yellow	
Yellow		

↑(Spinner 2 options)

Counting Tree:



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4. Explain that using any one of these sample spaces is acceptable.
5. As a class, walk through together making a tree for making purple.
6. Then let the students work individually or in pairs to make a counting tree for the paper, rocks, scissors. Circulate and assist those who need help.

More Sample Space:

Materials: bag or opaque bucket,

2 red, 3 white, and 4 blue cubes

Steps:

1. In a bag, place 2 red, 3 white, and 4 blue cubes. Have each student draw out of the bag 1 cube, replace it in the bag and then record what color they drew out of the bag.
2. Each student makes a guess as to what colors are in the bag and how many of each.
3. Repeat steps 1 and 2 and second and third time.
4. Check everyone's guesses by showing them what was in the bag.
5. As a class or in pairs have the student's compute the classes empirical data for the following probabilities: P(drawing 2 white); P(not drawing blue); P(drawing red and blue); P(drawing red then blue)
6. Construct a sample space for drawing out of the bag to compute the following theoretical probabilities after 2 draws: P(drawing 2 white); P(not drawing blue); P(drawing red and blue); P(drawing red then blue)
7. Help students explore shortcuts they can use in making the counting tree or possibly other sample spaces they created.
8. Go over the theoretical probabilities as a class and ask groups to share how they found their information.

Even more Sample Spaces!

Using the same blocks as before, students should create a sample space, but do not return the first cube to the bag. Compute the same theoretical probabilities as before.

Lesson 6: Expected Value

Materials:

Steps:

1. Draw a circle on the board divided into 3 equal sections. Tell students that this is a game from the carnival. It costs $1 to play the game. You win the dollar amount on each section. Section 1 is worth $1, section 2 is $2, and section 3 is $3. Is this a fair game?
2. Tell students that you are going to find the expected value of the game or the long term average. We can compute expected value by using the following equation where E = expected value.

E = ($ of section 1) * (fraction of circle for 1) + ($ section 2)* (fraction of circle for 2) + ($ section 3) * (fraction of circle for 3) – the amount charged to play the game

Ex. $1 (1/3) + $2 (1/3) + $3 (1/3) - $1 = $1 In other words, you will win on average $1 every time you play.

Share with students that games that have an expected value of $0 is fair, games with a positive expected value are in favor of the person playing, and games with an expected value that is negative is in favor of the person running the game.

3. Give students a circle divided into 4 equal sections: 1 section labeled with $1, 2 sections labeled with $2 and 1 section labeled $1. Tell them this game costs $2 to play. Find the expected value of this game.
4. Give students time to work and go over the steps again as a class.
5. Repeat process giving the students circles or squares divided into various sections worth various dollar amounts.

More Expected Value Problems:

Put the 4 following games on the board and have the students rank them in order from 1-4 on what they think are fair games (1 = most fair; 4 = least fair) After students have guesses recorded in partners or on own have them find the expected value of each game.

1. Deck of 52 cards

   Cost $1 to play

   If you draw a Spade you win $50 all others cards you win $0.

   Find the expected value.

2. Flip 2 Coins

Cost $2 to play

If you draw 2 heads you win $2

All other turns you win $0.50

Find the expected value

3. Deck of 52 cards

   Cost $10 to play

   Draw an Ace you win $20

   Draw a face card win $15

   Draw any other card and you win $5

   Find the expected value

4. Bag of 1 red, 2 white, and 3 blue cubes

   Cost $2

   Draw red win $3

   Draw white win $2

   Draw blue win $1

   Find the expected value

Can also go back and try to find the expected value of Match/No-Match and Paper, Rock, Scissors if students need more practice or as an assessment at this point as well.

PERFORMANCE PACKAGE TASK 1

Carnival Games

Content Standard: NCTM Data Handling, Statistics and Probability **Level:**Grades 7-8

Specific Statement(s) from the Standard:

- Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations
- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models

Product(s):

Develop a game to be used in a classroom carnival

Task Description:

Students will each design a carnival game keeping the following items in mind:

- The game should make a profit for the school
- Game should be easy to set up and inexpensive
- Game should take a short amount of time to play
- Rules of the game should be clearly written and easily understood.

Students will write a report of the game they designed and include the following:

- Complete set of rules for the game including how to play, how much it costs to play, and how much a player wins.
- Explain how they determined the theoretical probability of winning the game as well as a sample space.
- Explain how they calculated the expected value of the game and the equation they used.
- Explanation of why they think people will want to play their game.
- Data collection sheet of outcomes
- Comparison of empirical data to theoretical expected values.

Special Notes:

PERFORMANCE PACKAGE TASK 1

(Title of Package)

FEEDBACK CHECKLIST FOR TASK 1

The purpose of the checklist is to provide feedback to the student about his/her work relative to the content standard. Have the standard available for reference.

Y=Yes

N=Needs Improvement

Student		Teacher
_____	Game is approved by the instructor.	_____
_____	Develop a game board that is visually appealing and appropriate.	_____
_____	Rules of the game state how much the game costs to play, how much a player wins, and is posted near playing area.	_____
_____	Students prepared appropriate data collection sheets.	_____
_____	Accurately calculated theoretical probabilities and expected values so that it is in favor of the school	_____
_____	Students compared the empirical data to theoretical expectations	_____
_____	Students mathematically reasoned why other students would want to play their game.	_____

Overall Comments (information about student progress, quality of the work, next steps for teacher and student, needed adjustments in the teaching and learning processes, and problems to be addressed):