Geometry Unit

Constructing Points of Concurrency

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Disclaimer: Many activity ideas are from "Discovering Geometry" materials by Key Curriculum Press.

Unit Objectives

- Construct the midpoint of a segment using a compass and straight edge.
- Construct the perpendicular bisector of a segment using a compass and straight edge.
- Construct perpendicular lines to another line using a compass and straight edge.
- Construct angle bisectors using a compass and straight edge.
- Construct the perpendicular bisectors of the sides of triangles, and make conjectures.
- Construct the angle bisectors of a triangle, and make conjectures.
- Construct the medians of a triangle, and make conjectures.
- Construct the altitudes of a triangle, and make conjectures.
- Identify the circumcenter and construct the Circumcircle of a triangle.
- Identify the incenter and construct the Inscribed circle of a triangle.
- Identify the centroid.
- Identify the orthocenter.
- Construct the Euler Line and identify which of the points of concurrency it runs through.
- Solve real-world problems using what you have learned about constructions and points of concurrency, and do so without knowing in advance what the intended model for solving is.

Lesson Plan - **Day 1**

Objectives:

- Construct the midpoint of a segment using a compass and straight edge.
- Construct the perpendicular bisector of a segment using a compass and straight edge.

Starter: Discuss what it means to be a midpoint, what are some midpoints in the real world, and how you would/could find the midpoint of a segment.

Activity: Using only a compass and a straight-edge, have students in groups of four construct the midpoint of a segment. When each group has had appropriate time (5 – 10 minutes?), have groups put their solution on the board. Discuss the solution and how to get it for another 5 minutes, and introduce/reiterate any vocabulary involved.

Discussion and/or Conjectures: Groups demonstrate on the board (in their area) how they arrived at their construction of a midpoint, teacher circulates to facilitate and evaluate. The teacher will bring the classes' attention back together and will either have a group summarize or the teacher will walk through it with everyone to make sure all have understanding.

Starter: Discuss what it means to be perpendicular.

Activity: Have groups explore a way to construct a perpendicular line through the midpoint. If/when a group thinks they have a solution, then they should put it on the board. If necessary, get them started on their way by having them explore circles from each end point.

Discussion and/or Conjectures: The teacher will bring the classes' attention back together and will either have a group summarize or the teacher will walk through it with everyone to make sure all have understanding.

Assignment: Construct the midpoint and perpendicular bisector of three different line segments (you pick the segments), and show your marks. Also, solve the following problem: There are two schools on an island in the pacific, and the superintendent wants to divide the island into two zones so that anyone within each zone is always closer to their own school. If a sheet of graph paper roughly estimates the island shape, and with the origin in the middle of the graph paper representing the middle of the island, the two schools are located at (8,12) and (-12,-11). Locate the dividing line between the two zones. Explain how you know this dividing line solves the problem. Or, pick several points in each zone and make sure they are closer to that zone's school than they are to the other one.

Lesson Plan – **Day 2**

Objectives:

- Construct perpendicular lines to another line using a compass and straight edge.

Starter (Case 1: Point on a line.): Walk around and look at students' effort on their homework while giving students opportunity to explain their solution to the "school" problem from yesterday. Once everyone is in understanding of this solution, pose the problem of finding the perpendicular to a line through a point on the line. Discuss what challenge(s) there might be with trying to use the method of finding the perpendicular bisector of a segment, which we have already learned.

Activity: Have each group find a way to construct the perpendicular to a line through a point on the line, and put their solution on the board. Have one group present and answer questions until everyone is satisfied with the solution.

Starter (Case 2: Point off a line.): Pose the question of constructing a line perpendicular to a line that goes through a point not on the line. Discuss what challenges there are in this case.

Activity: Have each group find a way to construct the perpendicular to a line through a point not on the line, and put their solution on the board. Have one group present and answer questions until everyone is satisfied with the solution.

Assignment: Give students two each of these constructions to do for homework. Also assign students explore doing both constructions with a piece of patty paper.

Lesson Plan – **Day 3**

Objectives:

- Understand shortest distance from a point to a line.
- Construct angle bisectors using a compass and straight edge.

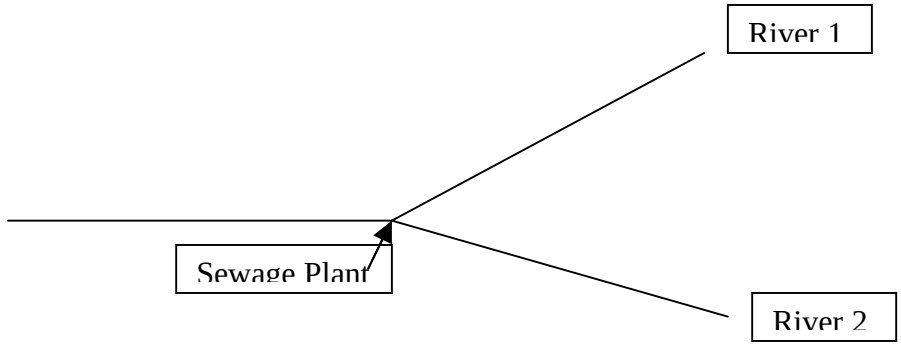
Startup: Layout the situation of a mouse in the room, who suddenly sees a cat hot in pursuit. If the mouse can make it to the wall of cabinets before the cat catches him, he will be safe (he can get under them, and the cat can't). Draw a sketch of this on the board and ask students to think about exactly what path the mouse should take to the cabinets, as he tries to survive.

Activity: Use Geometer's Sketchpad to explore the concept of the shortest distance from a point to a line, and have students write a short explanation of the best path for the cat to take and why. As a whole group, form a conjecture about the shortest distance from a point to a line.

Startup: Pose the question of constructing the bisector of an angle. Ask the students what construction that we are familiar with might help in this construction. Once the perpendicular bisector construction is mentioned (by you if need be), send them into their groups for the activity.

Activity: Have each group find a way to construct an angle bisector, and put their solution on the board. Circulate to keep groups on track. Have one group present and answer questions until everyone is satisfied with the solution.

Assignment: Have students show the shortest path from a point to a line on a few graphs, and have them do two angle bisector constructions. Also have them solve the following problem: There is a sewage treatment plant at the point where two rivers meet. You want to build a house near the two rivers (upstream, naturally, from the sewage plant), but you want the house to be at least 5 miles from the sewage plant. You visit each of the rivers to go fishing about the same number of times, but, being lazy, you want to minimize the amount of walking you do. (You want the sum of the distances from your house to the two rivers to be minimal.) Where would you build your house? Here is a model of the rivers and plant. Reconstruct this model in a new sketch, and investigate possible locations for the house.



Lesson Plan – **Days 4, 5, and 6**

Objectives:

- Construct the perpendicular bisectors of the sides of triangles, and make conjectures.
- Construct the angle bisectors of a triangle, and make conjectures.
- Construct the medians of a triangle, and make conjectures.
- Construct the altitudes of a triangle, and make conjectures.

Startup: Review with students the constructions they have learned the last three days. Also, give an informal proof of why the best place to locate the house in the river problem is on the bank of one of the two rivers. For today's assignment, ask them for a definition of an altitude of a triangle and the median of a triangle.

Activity: Have each group use "Geometer's Sketch Pad" to construct the three perpendicular bisectors of a triangle, and make any conjectures they can. Circulate to keep groups on track. Have one group present and answer questions until everyone is satisfied with the final conjectures.

Activity: Have each group use "Geometer's Sketch Pad" to construct the three angle bisectors of a triangle, and make any conjectures they can. Circulate to keep groups on track. Have one group present and answer questions until everyone is satisfied with the final conjectures.

Activity: Have each group use "Geometer's Sketch Pad" to construct the three altitudes of a triangle, and make any conjectures they can. Circulate to keep groups on track. Have one group present and answer questions until everyone is satisfied with the final conjectures.

Activity: Have each group use "Geometer's Sketch Pad" to construct the three medians of a triangle, and make any conjectures they can. Circulate to keep groups on track. Have one group present and answer questions until everyone is satisfied with the final conjectures.

Assignment (Day 4): Construct the three perpendicular bisectors of an obtuse triangle using a straightedge and a compass.

Assignment (Day 5): Construct the three angle bisectors of an obtuse triangle using a straightedge and a compass. Also construct the three medians of a triangle using a straightedge and a compass.

Assignment (Day 6): Construct the three altitudes of an obtuse triangle using a straightedge and a compass.

Lesson Plan – **Day 7**

Objectives:

- Identify the circumcenter and construct the Circumcircle of a triangle.
- Identify the incenter and construct the Inscribed circle of a triangle.

Startup: Review with students two of the constructions they have learned the last three days. Discuss with them how interesting it is that the three perpendicular bisectors intersect at a point, and that the three angle bisectors intersect at a point. Remind them of the conjectures they came up with concerning these constructions, and note that it might be nice to associate each of these "Points of Concurrency" (define) with a name (define).

Activity: Second chance conjectures: Have groups take another look at both the perpendicular bisector and angle bisector constructions to see if we have missed any conjectures. After a quick ten minutes, discuss any possible conjectures each group might have come up with. Finally, define incenter and circumcenter. Using past homework assignments, have students draw the inscribed and circumscribed circles as appropriate. Circulate to make sure they are having success.

Rigor: Give informal proof of why the incenter, circumcenter are equidistant from the sides and vertices respectively.

Assignment: Given two congruent triangles each, have students construct both a circumscribed and an inscribed circle, respectively.

Lesson Plan – **Day 8**

Objectives:

- Identify the centroid.
- Identify the orthocenter.

Startup: Review with students how interesting it is that the three medians of a triangle intersect at a point, and that the three altitudes of a triangle intersect at a point. Remind them of the conjectures they came up with concerning these constructions, and note that it might be nice to associate each of these "Points of Concurrency" with a name (define).

Activity: Second chance conjectures: Have groups take another look at both the median and altitude constructions to see if we have missed any conjectures. After a quick ten minutes, discuss any possible conjectures each group might have come up with. Have the groups construct the medians of a triangle on Geometer's Sketchpad and check each of these possible conjectures.

Assignment: Given two congruent triangles each (same exact triangles as yesterday), have students construct both a centroid and an orthocenter. Again using see-through paper and being extremely neat. Also using marker for the triangle sides and the point of concurrency.

Lesson Plan – **Day 9**

Objectives:

- Construct the midpoint of a segment using a compass and straight edge.
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- Construct angle bisectors using a compass and straight edge.
- Construct the perpendicular bisectors of the sides of triangles, and make conjectures.
- Construct the angle bisectors of a triangle, and make conjectures.
- Construct the medians of a triangle, and make conjectures.
- Construct the altitudes of a triangle, and make conjectures.
- Identify the circumcenter and construct the Circumcircle of a triangle.
- Identify the incenter and construct the Inscribed circle of a triangle.
- Identify the centroid.
- Identify the orthocenter.
- Construct the Euler Line and identify which of the points of concurrency it runs through.
- Solve real-world problems using what you have learned about constructions and points of concurrency, and do so without knowing in advance what the intended model for solving is.

Startup: Question students as a whole group on what constructions we are able to make, and list them on the board. Ask them to work with their point of concurrency constructions to see if there is something "interesting" to learn when they put all four constructions together (remember, it is see-through paper).

Activity: Students explore their combined constructions. Teacher leads a discussion after about 5 minutes to bring out what they found (Euler line hopefully) and which points are on it.

Group assignment: Construct another Euler line. Each of the four group members are assigned one of the points of concurrency to construct, with all members having a copy of the same obtuse triangle. When done, they should put them together to find the Euler line.

Lesson Plan - **Day 10**

Final Project – Individual
Construct all the points of concurrency and the Euler line. Teacher will hand out congruent triangles – one to each student. Using different colors in their compass for each construction, students will then add the Euler line. This project will allow students to demonstrate each of the unit objectives on an individual basis.