Archimedes' Calculation of *Pi*
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Part I: Lesson Plans

Day One: Introduction

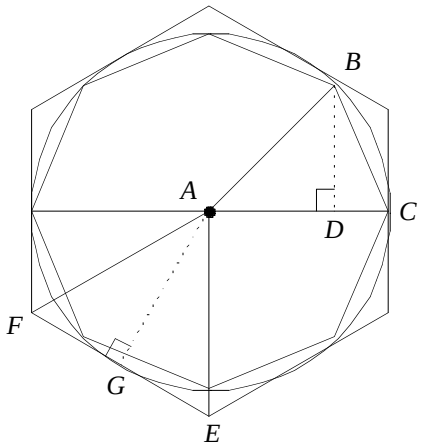
Target Learners: Second semester geometry students
Student Materials: Scientific or graphing calculators
Instructor Materials: Overhead projection system

Objectives: Upon completion of this lesson, the student will be able to:

Calculate the approximate value of *pi* using the ancient Egyptian method.
Calculate the approximate value of *pi* using Plato's method.

Overview of Lesson: Open the unit by discussing that calculating the value of *pi* has been performed by mathematicians since ancient times. Briefly discuss the background of Egyptian mathematics, their emphasis on application problems involving measurement, and various conjectures on how they developed their mathematics. Introduce the premise that ancient Egyptians believed that the area of a circle was approximately equal to the area of the square whose side was $\frac{8}{9}d$, where *d* is the diameter of the circle. The students will be asked to set up and solve the equation $\pi r^2 = \left(\frac{8}{9}d\right)^2$ to find the approximate value of *pi* (3.16) used by the Egyptians.

Continue the discussion by presenting Plato's belief that all of nature was based on the four elements of fire, air, water, and earth, and these elements could be represented by the polyhedra tetrahedron, octahedron, icosahedron, and cube.
According to Plato, these polyhedra were all definable by three types of line segments: commensurable segments of a given unit of length, the diagonal of a square with a length of $\sqrt{2}$, and the altitude of an equilateral triangle, whose length would be $\sqrt{3}$ for a side length of the given unit length. Plato investigated the area of a circle, believing that the value of *pi* was the average of the areas of an octagon inscribed in a circle and a hexagon circumscribed about the circle.



The students will be asked to calculate the area of *pi* using Plato's method, using the figure to the right as a guide.
The radius of circle *A*, $\overline{AC}$, will be assigned the value of one unit.

The conclusion will be that Plato believed that *pi* was the average of $2\sqrt{2}$ (the area of the inscribed octagon) and $2\sqrt{3}$ (the area of the circumscribed hexagon), or $\sqrt{2}+\sqrt{3}$. This value is approximately 3.14626.

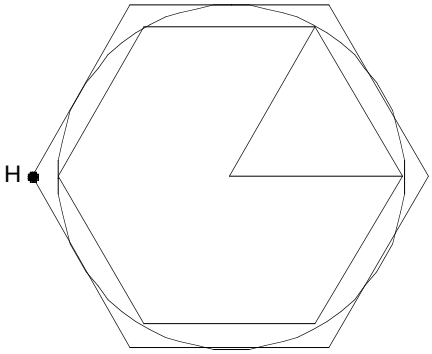
Day Two: Archimedes' Method – Part I

Student Materials: Access to Geometer's Sketchpad software
Instructor Materials: Overhead projection system, Geometer's Sketchpad software

Objectives: Upon completion of this lesson, the student will be able to:

- Construct inscribed and circumscribed hexagons and dodecagons using Geometer's Sketchpad.
- Use Sketchpad's built-in calculator to find the perimeter of inscribed and circumscribed polygons.
- Calculate the approximate value of *pi* using the data from Sketchpad and Archimedes' method.

Overview of Lesson: The second day's work will open with a discussion about the ancient Greek mathematician Archimedes and his many contributions to mathematics, including his treatise on the measurement of a circle. His method of calculating the value of *pi* by using a method of exhaustion involving the perimeters of regular polygons inscribed in and circumscribed about a circle will be introduced. Student activities will take place in a computer lab where students can access Geometer's Sketchpad. They will be asked to construct a circle, and then inscribe one hexagon inside the circle and circumscribe a second hexagon about the circle. They will use the construction to calculate the approximate value of *pi* using Archimedes' method. A students' Sketchpad work will look something like the figure below.



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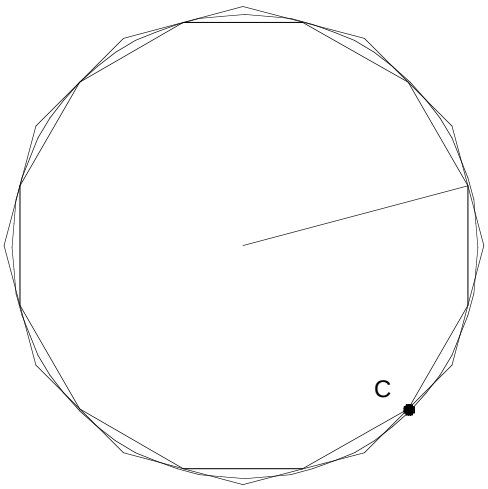
Following Archimedes' train of thought and method of exhaustion, the students will construct a second figure consisting of a circle with inscribed and circumscribed dodecagons and repeat the process of calculating the approximate value of *pi* using Archimedes' method. The following example shows what a student's work might look like.



12˙NO

2˙MA

<u>Day Three</u>: Archimedes' Method – Part II

Student Materials: Scientific or graphing calculators
Instructor Materials: Overhead projection system

<u>Objectives</u>: Upon completion of this lesson, the student will be able to:

- Find the lengths of the sides for each of the regular 48-gons that are inscribed in and circumscribed about a unit circle.
- Find the perimeters of the regular 48-gons that are inscribed in and circumscribed about a unit circle.
- Calculate the approximate value of *pi* based on the perimeters of inscribed and circumscribed 48-gons using Archimedes' method.

<u>Overview of Lesson</u>: Calling upon their work and findings from their work using Geometer's Sketchpad, the students will determine how to find the approximate value of *pi* reached by Archimedes using a regular 48-gon. The students will begin by finding the length of one side of a regular 48-gon inscribed in a unit circle. This will require finding the measure of the vertex angle (central angle) for the isosceles triangle that has a side of the 48-gon as its base. Then, using the Law of Sines, the students can find the length of the base. The same process is used to find the length of a side of the circumscribed 48-gon.

Once the side lengths for the two 48-gons are found, the perimeters can be calculated and used to find Archimedes' approximate value of *pi*.

To conclude the unit, the students will be asked to demonstrate their understanding of Archimedes' method of calculating *pi* by finding the approximate value of *pi* using the perimeters of inscribed and circumscribed regular 96-gons. In addition to their calculations, the students will be asked to explain, in writing, Archimedes' method and how they found their solution. Four or five students will be asked share their solutions and explanations with the class.

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Part II: Task Description

Meeting the Standards

This project meets several of the standards found in the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* published in 2000. Specifically, this project should facilitate the students meeting the following standards:

1. Communication Standard for grades 9-12 (p. 348)

   Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
   Analyze and evaluate the mathematical thinking of others; and
   Use the language of mathematics to express mathematical ideas precisely.

   This project gives the students the opportunity to simulate the mathematicians who discovered and developed the concepts in use today. The students share their findings with their peers and teacher through the presentation of their work. According to the *Principles and Standards* (2000), "To be prepared for the future, high school students must be able to exchange mathematical ideas effectively with others" (p. 349).

2. Connections Standard for grades 9-12 (p. 354)

   Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

   By experiencing various methods for calculating *pi* from the past several thousand years, the students will gain a feel for how mathematical concepts were discovered and how they have connected and built upon each other. Many past mathematicians have also been scientists, astronomers, and so on, using their mathematical knowledge and skills in these other areas. As the *Principles and Standards* (2000) say, "When students can see the connections across content areas, they develop a view of mathematics as an integrated whole" (p. 355).

3. Geometry Standard for grades 9-12 (p. 308)

   Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

   Specifically, the expectations for this standard are that all students should

   analyze properties and determine attributes of two- and three-dimensional objects;
   explore relationships (including congruence and similarity) among classes of two- and three dimensional objects, make and test conjectures about them, and solve problems involving them;
   establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others; and
   use trigonometric ratios to determine lengths and angle measures.

Products

Upon completion of the unit, each student will be able to calculate *pi* using Archimedes' method for a pair of inscribed and circumscribed 96-gons. In addition, he or she will be asked to describe Archimedes' method and explain his or her solution in writing.

Task Description

Based on their previous work and findings, particularly their work with the inscribed and circumscribed 48-gons, the students will be given the opportunity to demonstrate their understanding of Archimedes' method for finding the value of *pi* by calculating the value using inscribed and circumscribed 96-gons. To extend their demonstration, they will be asked to explain the method and their solution for the 96-gon in written form.

To conclude the unit, four or five students will be chosen at random to share their solution and explanation with the rest of the class.

Special Notes

References

Cooke, R. (1997). *The history of mathematics: A brief course*. New York: John Wiley & Sons, Inc.

Dunham, W. (1991). *Journey through genius: The great theorems of mathematics*. New York: Penguin Books.

Principles and standards for school mathematics. (2000). Reston, VA: National Council of Teachers of Mathematics.

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Part III: Feedback Checklist

_____
Student's Name

S = Satisfactory
N = Needs Improvement

Student		Teacher
_____	Calculate the approximate value of *pi* for a 96-gon using Archimedes' method.	_____
_____	In writing, describe Archimedes' method for calculating *pi* and the solution for *pi* based on the inscribed and circumscribed 96-gons.	_____

Additional Comments: