Probability Lesson Plans Grades 9-11

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Day 1 Finding sample spaces

Note: Have students keep all work throughout unit accessible.

Small groups (3 or 4), each group needs two different colored markers or colored pencils and a handout with the same face missing a mouth repeated several times.

Problem 1: In their groups, students should demonstrate all possibilities for a face. Options include two different colors (one per face) and three shapes for the mouth (smile, frown, or straight line). When they are finished, they should answer the questions, "How many different faces did they find?" and "How could they represent this as a list?".

Problem 2: In their groups, students should demonstrate all possibilities for a face with one set of additions. The new addition is two types of hair (spiky, curly). Again they should answer the questions, "How many different faces did they find?" and "How could they represent this as a list?".

Have student groups share with the entire class methods of finding/representing a list of the sample space (i.e. list, table, tree).

Problem 3: In their groups, students should find a way to represent the sample space for the following problem. You are at the concession stand during a basketball game. You are buying three different things (a drink, a fruity treat, and a chocolate treat). Of the drinks, your choices are juice (J), pop (P), and water (W). Of the fruity treats, your choices are skittles (S) and licorice (L). Of the chocolate treats, you have a choice of M&M's (M) and Hershey's Bar (H). What are all the different purchases you can make?

As a class, share the different ways of showing this sample space. Also talk about the terminology of sample space and choices, using the previous 3 examples.

Homework: Have the students bring home the following problem: Your parents are buying you a car for your 16^{th} birthday. How many cars are possible if your choices include 2 models (mustang and hummer), 3 color choices (red, black, and yellow), and 2 types of seat covers (leather and cloth). You may use any method of choice to find all possibilities.

Day 2 Finding sample spaces (continued)

Have students share with each other in their small groups how they found the sample space. In sharing, each group should have all three methods represented, if not, they should work together to find the missing method(s).

Problem 1: In their groups, students should answer the following question: You are planning your morning schedule for next year. Following are your options: 1^{st} hour (Biology, Chemistry, Art, Geometry), 2^{nd} hour (Band, Choir, English), and 3^{rd} hour (History or Health). What are all the possibilities for your morning schedule? How many possibilities are there?

Problem 2: Continue in groups. Now that your morning schedule is set, you are trying to plan your afternoon schedule. Following are your options: 4^{th} hour (Civics, PhyEd, Student Aide, Accounting, Computer), 5^{th} hour (Algebra II and American History), 6^{th} hour (Foods, Spanish,

and German), and 7th hour (Study Hall). What are all the possible afternoon schedules? How many possible afternoon schedules are there?

Have a class discussion about problems 1 and 2. Which method do the students prefer to find the sample space and why? What effect did 7th hour have on the sample space? Why? Does it matter what letters you pick for the choices in 5th hour? What about the letters you picked for choices between 6th and 7th hour?

Problem 3: Back to small group. You are voting on student council representatives. For president, Sally, John, Trudy, Mandi, Cole, and Bob are running. For Vice-President, Mark, George, Lindsey, and Paula are running. For Treasurer, Frank and Tami are running. What are all the possible councils? How many are there?

Homework: Find the sample space of the following problem 3 different ways: list, table, and tree diagram.

You are taking a test with three problems. The possible answers for question 1 are a, b, c, d. The possible answers for question 2 are true or false. The possible answers for question 3 are yes or no. Find the sample space of all the ways the test can be answered using 3 different ways. How many possible answer arrangements are there?

Day 3 Begin concept of Fundamental Counting Principle.

Collect homework. Have students work in groups to create a table of all of the problems covered so far. They should show how many choices there were for each event and how big the sample space was in the end. Make sure to explain that an event is the place where you have choices (example: question 1 on a test is an event, there are four choices, a, b, c, and d). They should work on answering the question, "Is there a way to find the size of the sample space without actually finding the sample space"? When the groups come up with an idea, they should test it with the following question by first trying the method they found and then actually finding the sample space.

If you have 5 shirts and 2 pairs of pants as options for clothing this morning, how many different outfits could you have worn?

Have the groups share with the class their ideas. Discuss the reason for multiplication in the Fundamental Counting Principle. Make sure to thoroughly investigate and study the smaller examples and the concept of multiplication. Remind the students of the afternoon schedule problem and discuss why 7th hour did not effect the size of the sample space. If possible, use that problem and the morning schedule problem to find how many full schedules there are possible, morning and afternoon. Also try a few more examples as a class and small group, making sure to cover the importance of what an event is and the number of choices involved in the event.

Homework: The homework is meant for the students to start intuitively thinking about factorial and to reinforce the Fundamental Counting Principle.

1. How many different cars can you have if you have the following options: 3 types of seats (leather, cloth, sheep skin), 5 colors (yellow, red, black, green, gold), 6 models (Malibu, mustang, geo metro, thunderbird, intrepid, and viper).

2. You are picking out an outfit for school. Your choices for shirts are red, blue, yellow, white, plaid, or green. Your choices for pants are jeans or khakis. Your choices for socks are black, blue, gray or white. How many outfits are possible?
3. You are going to make a 3-letter word. You can use any letter in the alphabet for each letter. How many words can be made assuming that any arrangement of letters is a word.
4. You and 2 of your friends race to the lunch line. How many ways can you stand in line? Give your friends names and show me the sample space.

Day 4 Factorial

Have students get into their groups and check their homework answers with each other on questions 1 and 2.

Give a short individual quiz on the Fundamental Counting Principle (1-2 problems). When everyone is finished, have students compare their answers from homework problems 3 and 4 in their groups. Have each group explain how they did their problems with the class. Discuss what the events are in these problems.

Have the groups try the following problems together. How many ways can you arrange the letters a, b, c, d and e? How many ways can 5 people finish a race (i.e. 1^{st}, 2^{nd}, etc)? How are those two problems related? What are the events in each problem?

As a class, discuss the answers to these problems. Form the definition of n! and permutations, expressing that it matters what order the events occur and that all the 'choices' are used. Reinforce with more problems.

Homework: Have students do three problems finding the size of the sample space that reinforce the idea of n!. (i.e. there are 6 different bottles, how many ways can they be arranged on a shelf). Also, have the students come up with their own example that has not already been used in previous examples. Their example should find the size of the sample space using n!

Day 5 Permutations

Have students work in groups to compare answers to their homework and also solve each other problem. Have a few groups share some problems with the rest of the class.

As a class, discuss how n! is related to the Fundamental Counting Principle learned earlier. How are they the same? How are they different? The idea is with n!, the choices are coming from the same place for the different events, and you are not replacing after using one choice. They are similar because you have a certain number of events and you are multiplying the number of choices for each event to find the size of the sample space.

In groups have the students work on the following problem: Six people are running a race. 1^{st}, 2^{nd}, and 3^{rd} place are the only places being recorded. How many outcomes for the race are possible? If the students are having difficulty, a hint would be to have them find the desired events (1^{st}, 2^{nd}, 3^{rd} place) and then figure out how many choices are possible for each event.

Discuss as a class how this problem works and how it is similar/different from n! Establish that this is still a permutation, you are just not using all of the choices to fill your events. Have the students practice on a few more problems.

Introduce the notation for permutation, ${}_nP_r$. If you use all of your choices, then r = n. Show this notation for the preceding examples.

Homework: Assign 6 – 8 problems that involve permutations where the students must know whether they use n! or 'cut it short'.

Day 6 Permutations (continued) Develop a formula for ${}_nP_r$

Have the students compare answers to homework in groups and then present answers on the board. Discuss that when all the choices are used, the formula is ${}_nP_n = n!$. In their groups, they should work on finding a formula for the problems where they do not use all of the choices (i.e. ${}_nP_r$). If they are having difficulty, they may want to make a chart for the homework problems where they list n, r, n! and r!.

Have the groups share with the class how they found their formula. Reinforce with a formal definition and formula. Discuss how this works for ${}_nP_n = n!$ and when r = 0.

Homework: Answer the following problems.

1. 9 girls are standing against a wall at a junior high dance. 2 boys ask 2 girls to dance. How many different couples could be dancing?
2. There are 8 different books, all the titles start with a different letter. There is only enough room for three books on the shelf. How many ways can the librarian put the books on the shelf?
3. There are 5 rides as a fair. How many ways can 5 people enjoy these rides if they can only ride one ride?

Day 7 Combinations

As the students enter the classroom, have the three homework problems on the board and room for the students to share their answers. Review each problem as a class. Collect the homework.

Pose the following problem and have the students get into groups to try to solve it. "You are buying flowers for you mom. At the flower shop, there is a bucket with 3 roses. You have decided to buy your mom 3 flowers. How many ways can you buy 3 flowers from the bucket of roses?"

Have the groups share with the class how they found their answer. How is this problem different than a permutation problem?

Have the groups explore the next problem:
"Three people are standing in line to see the same movie at the movie theatre. How many ways can they stand in line? If the movie is sold out and there are only two seats left, how many ways can two people get in to see the movie out of the three people standing in line?"
Students may want to make a sample space if they are having difficulty with the answer.

Have the groups share with the class how they found their answer. How is this problem different than a permutation problem? Is it the same difference as the first problem? Discuss ideas for how to find this new type of problem. Leave it just at ideas before moving on to the next problem (Don't tell them how to do it yet).

Day 8 Combinatons

Continuing from the previous day, have the groups work on the following questions, thinking about ways to find the answer by way of a formula. "The FACS class is raffling of two identical quilts. Four different people buy raffle tickets. How many ways can two people win the quilts?" Again, it may help to make a sample space.

Have the groups share with the class different ways of finding the solution. Is there a formula to this type of problem? Start discussing how the order does not matter and so there must be fewer ways this can happen, hence divide by a certain number. What number could that be and why?

Try another problem. "You and 4 friends go to the local convenience store to buy a mountain dew each. There are only 3 left. How many ways can you and your friends buy the pop? Assume you are not sharing them".

Discuss how to find the answer to this problem and in general how to find any of these problems using the formula for combinations. Why do you divide by r!? Introduce the word combinations and the notation used for it.

Homework: Assign 6-8 combination problems.

Day 9 Combinations

Have students review homework assignment with group. Collect homework.

Start working on putting it all together, FCP, permutations, and combinations.

- What is the big idea for combinations?
  - Notation and formula
  - Order does not matter
- What is the big idea for permutations?
  - Notation and formula
  - Order does matter
- What is the big idea for the Fundamental Counting Principle?
  - Certain events, each can happen a certain number of times
  - Multiply them together

BIG QUESTION: Does order matter?

Also discuss the inappropriate use of the word combination!

Bring in new problems of combinations and permutations. Pair them together so that each pair has the same story, but changes it enough to make it either a combination or permutation. Following is an example.

- You are participating in a contest for memorizing pi with 10 other students. The top three memorizers win a free pizza. How many ways can this happen?
- You are participating in a contest for memorizing pi with 10 other students. The top memorizer wins $50. The second top memorizer wins $25. And the third top memorizer wins $15. How many ways can this happen?

First have students find in which problem order matters. Then find the answers to the problems.

Homework: Assign two problems of each. Students should specify whether order matters and what the answers are to each, making sure to clearly show how they arrived at their answers.

Day 10 and 11 Permutations and Combinations

Have students compare their homework in their groups and then share with the class the method to their answers.

Give a short quiz on permutations and combinations. Make sure to label which method they should use. You may want to include a question as to why the student should use the specific method.

Keep giving the groups problems to work on together. Make sure to include Fundamental Counting Principle problems, besides just permutation and combination problems. Students should not only produce answers, but should show a clear reason for their answers. Following are examples of problems you can give them.

- A license plate has six spaces. The first three spaces are letters, the last three spaces are numbers. How many license plates are possible?
- You have 10 posters that you want to put on your wall in your room, but you only have 3 spaces you can put the posters. How many ways can you arrange the posters you have room to hang? (This problem has potential for discussion. Does it matter which poster is on the left and which is on the right?)
- There is a shortage of flu shots for the school. There are 12 people who want flu shots, but there are only 7 shots left. How many ways can these people get the flu shots?

Homework: This will be the last assignment before the unit assessment.

The students should create their own problems and find the answers:

- 1 Fundamental Counting Principle
- 1 combination (r smaller than n) and (n smaller than 15)
- 2 permutations (n smaller than 15)
    - ♣ r = n
    - ♣ r smaller than n

Day 12 Permutations and Combinations including assessment

Have students share their problems within their groups. Allow groups to share with the entire class.

Assessment: The assessment is a story that the students will read and answer the problems written throughout the story. The story should be tailored to each student's ability (i.e. insert the appropriate sized numbers in the blanks). The students can discuss the problems within their groups, but they should all have different answers because of the individualized numbers. Make sure to stress that the students will be assessed not only on their answers, but also on their explanation to their methods of finding their answers. Give an example to show the students what an excellent answer looks like and otherwise.

PERFORMANCE PACKAGE TASK 1
(Theme Park Assessment)

Content Standard: Data Analysis, Statistics and Probability **Level**: Partial

Specific Statement(s) from the Standard:

Minnesota Standards: Select and apply appropriate counting procedures to solve real-world and mathematical problems.
NCTM Standards: understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases;

Product(s):
Work in groups to answer the questions in the given story using the correct counting methods.

Task Description:
Read the following story and answer the questions within the reading.

You and ___ of your friends go to your local theme park. On the way there only ___ people can sit in the very back seat of the van. How many ways can this happen?

When you get to the park, you all run to the rollercoaster, trying to be the first in line. How many ways can you all line up for the ride? They allow only ___ on each roller coaster car at a time, how many ways can this happen in your group if there is only one car remaining?

After the roller coaster, you and your friends decide to play a game. You go to the booth called "Guess Your Luck". Here's how it works, the dealer is going to (flip a coin and roll a six-sided die)/(Flip 2 coins)/(roll two die)/(Flip 2 coins and roll a die). But before he/she does, you are supposed to guess what is going to happen. What are all the possible outcomes? How many outcomes are there for this game?

After the game, you and your friends decide to race go-carts (only you and your friends will be on the track). First place wins a free ride, second place wins a free game, and third place wins a free soda. How many outcomes are there? (if there are only ___ go-carts available for your group of friends?)

You are hungry for some ice cream, so you go to the Snack Shack. You can choose from a red, blue, (etc) bowl, vanilla, chocolate, strawberry, (etc) ice cream, and caramel, chocolate, (etc) toppings. What are all the possible ways you can order your ice cream? How many ways are there?

Special Notes:
Since the students can work in groups on this assessment, and because students are at different levels, the numbers can be chosen separately for each student according to the student's ability. The students should be encouraged to discuss the problems together, since the numbers are different, the answers will also be different.

PERFORMANCE PACKAGE TASK 1

FEEDBACK CHECKLIST FOR TASK 1
(Theme Park Assessment)

The purpose of the checklist is to provide feedback to the student about his/her work relative to the content standard. Have the standard available for reference.

Y=YES
N=Needs Improvement

Student		Teacher
_____	Quality explanation and answer to the question in the first paragraph.	_____
_____	Quality explanation and answer to the first question in the second paragraph.	_____
_____	Quality explanation and answer to the second question in the second paragraph.	_____
_____	Construct a tree-diagram to list all possible outcomes in the game in the third paragraph.	_____
_____	Quality explanation and answer (not including or using information from the tree diagram) for the question of how many outcomes in the third paragraph.	_____
_____	Quality explanation and answer to the questions in the fourth paragraph.	_____
_____	Construct a tree-diagram to list all possible outcomes for your ice cream snack in paragraph five.	_____
_____	Quality explanation and answer (not including or using information from the tree diagram) for the question of how many outcomes for an ice cream snack in the fifth paragraph.	_____

Overall Comments (information about student progress, quality of the work, next steps for teacher and student, needed adjustments in the teaching and learning processes, and problems to be addressed):