

# SUMMER FINAL PROJECT

## NUMBER SENSE JUNIOR COLLEGE –PRE ALGEBRA DEVELOPMENTAL COURSES

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## Executive Summary

These lessons are for a Pre Algebra class for Junior College Mathematics. This course is a developmental course where students need help with their basic skills.

The students were placed in this course as a result of their placement test. Their low score was caused by weak mathematical background in recent school mathematics or for some, being out of education for many years. Therefore this material, which is at the middle school level, should be a review or reminder for most students.

The main topics in this course are the study of Whole numbers, Integers, Fractions, decimals, percents, measurement, ratio and proportion and a little intro to Algebra.

These lessons will cover the first unit, Whole numbers and Integers. It will cover about 3 weeks. The class meets Monday, Wednesday, and Friday for 50 minutes. I have Office hours for 2 hours those days and 1 hour Tuesday and Thursdays.

Since most everyone in the class will have knowledge of the Algorithms for the operations of whole numbers, these lessons will concentrate on the number skills that will help them better understand how these Algorithms work. Most of the class time will be spent doing number sense investigations in groups. The students will have assignments to do outside of class from a traditional textbook on the basic skills. They will have group quizzes on these assignments and a unit test on them.

I will give this plan a try this fall in my two Pre Algebra classes.

## Lesson 1

**MN Standard addressed:** The student should be able understand numbers, ways of representing numbers and number systems. And to understand the meaning of operations and how they relate to each other.

**Topic** – Understanding whole numbers and the concept of place value.

1. I will put them in groups of 2 or 3 and issue each group some Base 10 blocks of 100's, 10's and units. I will put on the board some 2 or 3 digit numbers and ask them to illustrate them with the blocks. Examples 352, 77, 439, 58,105,14,3
2. Next, we will have them illustrate with the blocks another way of showing 105 and 14 with the blocks.
3. We move to addition and have them try some problems with the blocks.  $35 + 44$  ,  $35 + 94$ ,  $35 + 178$ ,  $352 + 439$
4. Next do some subtraction problems  $44 - 23$ ,  $44 - 35$ ,  $178 - 125$ ,  $405 - 137$

Finally I will load on to the computer the place value game found on the National Library of Virtual Manipulatives – Utah State University for addition and subtraction. This has great visual demonstrations of how place value affects these operations. To get to this site

1. type in NLVM.com
2. select The Five Towns.com
3. follow to Welcome to the Four Towns Directory 4 Kids
4. Select National Library Virtual Manipulatives
5. A grid will show, select Numbers and Operations for 6-8 grade box.
6. Select either addition or subtraction operations, and sample problems will appear in the Base 10 format. For addition you must encircle groups of 10, and they will automatically be put together as one block. Then you can move them to the next place value. For subtraction, when borrowing you must move a block from higher to lower and it will split apart so you can do the subtraction. Have the students come to the computer and try some operations.
7. Some number sense problems which show the ability to make intelligent decisions based on mathematical relationships and the context of these relationships.

Try the following problems and fit the numbers in the appropriate

1. 1801 3 1743 58 13

Thomas Jefferson was born in Virginia on \_\_\_\_\_ April \_\_\_\_\_. He was the author of the Declaration of Independence. He was also an architect, a scientist, and a farmer. In \_\_\_\_\_, at the age of \_\_\_\_\_, Thomas Jefferson became the \_\_\_\_\_ d president of the United States.

2. 27 1 206 54 26

Did you know that each of your hands has \_\_\_\_\_ bones? That means that you have a total of \_\_\_\_\_ hand bones. Your foot has \_\_\_\_\_ fewer bones than your hand, or \_\_\_\_\_ bones. Your body has a total of \_\_\_\_\_ bones.

3. 268 34 4200 50 84 10

A high school basketball court is \_\_\_\_\_ feet long and \_\_\_\_\_ feet wide. Its length is \_\_\_\_\_ feet more than its width. The perimeter of the court is \_\_\_\_\_ feet, and the area of the court is \_\_\_\_\_ square feet. The rim of the basket is \_\_\_\_\_ feet from the floor.

**SUMMARY:** Today I hope you learned how place value impacts addition and subtraction and that it helps you improve those skills.

Source: Carole Greenes, Linda Schulman, and Rika Spungin

## Lesson 2

Continuing the same standards as in Lesson 1

For reviewing Multiplication we will start with the game

Tic Tac Time Game --Product Game Investigation #1

Factors:        1        2        3        4        5        6        7        8        9

Game board

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Instructions: The students will be in pairs and they will need one board and each will have different colored tokens. The object is to get 3 tokens in a row either vertically, horizontally or diagonally. One token each will be used as a factor marker and the remainder as game tokens.

Example: player one puts a token on the factor line on a number say 4, player two puts his on 6. The product of these two is 24 and player two puts one of his tokens on the game board 24. Factor tokens may be placed on the same factor, resulting in squared numbers. Now player one can move his token from the 4 on to any number that will result in a position he desires on the game board to help him get 3 in a row or block player 2 from doing the same. Making an incorrect product will result in a penalty, where the opposing player can move both factor tokens anywhere he desires.

Source: Marilyn J Sweeney , Skyline Middle School, Wilmington,DE 19808

The next investigation #2 which helps with the understanding of place value in multiplication and helps with good number sense is solving the following for the missing letters.

$$\begin{array}{r} \text{S T R A W} \\ \times \quad 4 \\ \hline \text{W A R T S} \end{array}$$

You must think  $4 \times$  Some number W will give you S and so on until the number for STRAW and WARTS gives a true multiplication product.

Investigation #3 for the lesson is playing the Hangman Multiplication game.

$$\begin{array}{r} \phantom{X} \phantom{00} \phantom{00} \\ X \phantom{00} \phantom{00} \\ \hline \phantom{00} \phantom{00} \phantom{00} \\ \phantom{00} \phantom{00} \phantom{00} \\ \hline \phantom{00} \phantom{00} \phantom{00} \phantom{00} \end{array}$$

One person creates a 2 digit by 2 digit multiplication problem works it out on a paper and the other person tries to guess where each digit goes. The person guessing has to pick a digit and tell what column it falls in. If he misses a part of the hangman is developed (before starting, design your hangman, and agree on how many steps there are to complete it). If he gets it correct, that number is put in the correct place or places if it occurs in more than once in that column.

Source: Greg Sarles Bemidji State U.

The last investigation for this is a take home optional problem. With extra points for high totals. If 12 people hand it in the highest will get 12 points, the second high 11 points, and so on down to 1.

The game is “HOW GRAND IS YOUR TOTAL”

Using the board you can only use each digit 4 times through out the board to create the largest total. See the next page for the game board.

**SUMMARY:** These investigations were meant to help you understand how place value plays an important role in multiplication of whole numbers.



### Lesson 3

MN Standard : Understanding operations and how they relate to one another.

This lesson investigates the operation of division.

1. First we must define the meaning of divisibility and its relationship to multiplication. Also, establish the general rule for divisibility of the numbers 2,3, and 5. At this point I will review the meaning of ‘odd’ and ‘even’ numbers.

See if the students can give a definition for finding out when they can tell if a number is divisible by 2,3, or 5.

2. Using  $18/7$  as an example relate this to the traditional division algorithm box, such as used in long division. Set up the following equation

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

And see if anyone can transform the  $18/7$  problem to fit the blanks in the above equation. Then, see if they can establish which is the quotient, divisor, dividend and remainder.

3. See if any student remembers the definition of a Prime number.

Once the definition is established, show how to make a Prime factor tree for any number such as 48 or 45. Point out that this method will be used many times in this course.

4. It would be advisable to review a few examples of the long division algorithm, such as  $4756 / 57$ . This will cause problems for some and tell them you will help them individually with this division.

5. For a group investigation into divisibility, have them go through the ‘Sieve of Eratosthenes’ for the numbers up through 60. Explain the first step of crossing out the numbers divisible by 2, except for the number two. Have them then try the number three and so on. See if anybody can establish what the numbers not crossed out represent.

6. For a take home number sense puzzle problem, which takes some excellent thinking- pass out the “Flashlight Problem” given on the next page. And if anyone can figure it out, let them explain it to the class at the next session.

Announcement- at the end of next class will be a short readiness quiz from your first 3 textbook assignments.

**SUMMARY LESSON 3-** In this lesson you should have learned about divisibility, prime numbers and the relationship of multiplication and division.

## THE FLASHLIGHT PROBLEM

The Beatles have only 17 minutes left to get to their van so they can make it to their next concert. To get to their van, they have to cross a narrow bridge that only two of them can be on at a time. To make matters even worse, it is pitch black out and they only have a single flashlight. Any party of two who crosses the bridge must have the flashlight with them. The flashlight must be walked across the bridge; it cannot be thrown.

It takes each of the Beatles a different amount of time to cross the bridge, as noted:

Paul's rate:	1 minute
John's rate	2 minutes
Ringo's rate	5 minutes
George's rate	10 minutes

Any pair of Beatles who walk across must walk at the slower one's pace. For example, if John and Ringo walked across together, it would take 5 minutes.

Explain how the 'Fab-Four' can get across the bridge in the allotted 17 minutes in order to make it on time for their next concert.

No tricks are used to solve the problem. You can't have them swim, drive the van over and turn on the headlights, ride on each other's shoulders, build a new bridge, or any other tom-foolery. Just give it a try and think outside the ?

Source: Greg Sarles, Bemidji State University

## Lesson 4

MN standard: Understand the meaning of operations and how they relate to each other.

This lesson is to understand the order of operations if more than one occur in the same problem. These rules apply to not only whole numbers, but to all phases of levels of mathematics. They will appear in most problems you will occur in Math, Physics, Chemistry, Nursing, Business ect.

1. Understand the grouping symbols, parenthesis, Brackets, and the division line as used in fractions.

2. Review exponents  $3^2$ ,  $3^0$ ,  $3^{-2}$

3. Review Radicals mainly perfect squares, perfect cubes.

4. Now put to together the rules for operations for example a problem like  $2(8 \times 5 - 36) \div 3 + 15/3$

1. Do what's in the grouping symbols first in this case inside the parenthesis. That means simplify inside the parenthesis.

2. Next apply any exponents.

3. Multiplication and Division are the next in the order and if both are in the problem after doing steps 1 and 2, you do them in order from left to right.

4. Finally you do Addition or Subtraction. Again if both are in the same problem, you do them in order from left to right.

For the class number sense investigation which will be a good application we will use the 1-2-3-4 Challenge Work Sheet, which follows on the next page.

The text assignment has some excellent problems on order of operations. Remember to check your answers in the back of the text.

The last 15 minutes will be for Quiz #1 –the first 10 minutes will be done solo, the last 5 minutes together in groups to verify results.

Each individual must turn in a quiz with work shown.

**SUMMARY LESSON 4** – While learning the important rules of order of operation, we also covered definition of exponents and of simple radicals mainly perfect squares and perfect cubes.

1-2-3-4 – Challenge

Use all four numbers: 1,2,3,4 only once

And any of the 4 operations +,-,x,/ and exponents to  
Form expressions for

1 =

5 =

9 =

2 =

6 =

10 =

3 =

7 =

11 =

4 =

8 =

12 =

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## **Lesson 5**

MN Standard: Students understanding of operations with whole numbers and integers can also be enhanced as they examine the validity and utility of properties of operations, such as commutative and associative properties of addition and multiplication. Also, at this level we will look at the Identity, Inverse, and Closure properties. Knowing these properties is not a major topic at this point. We will run in them at various times in this class and they will become familiar to you then.

1. Hand back Quiz #1 and answer questions about it.
2. Go over an example of each property with respect to each addition and multiplication.
3. Hand out Investigation on Properties to be worked in groups of 3.

Make sure you can justify you answers with example to the properties Investigation. Either a positive or a negative example to justify your answer.

Properties Investigation

1. Complete the tables not completed and answer the questions, Yes or No(circle Y or N ) as to whether the property named applies in each case.

X	1	2	3	4	...	+	1	2	3	4	...	properties for +
1						1						C Y or N
2			6			2		5				A Y or N
3						3						Id Y or N
4						4						Iv Y or N
.						.						Cl Y or N
.						.						
.						.						

Properties for above table x

- C Y or N
- A Y or N
- Id Y or N
- Iv Y or N
- Cl Y or N

*	A	B	C	D
A	C	B	A	D
B	B	A	D	C
C	A	D	C	B
D	D	C	B	A

#	H	E	L	P
H	P	L	H	E
E	L	P	E	L
L	E	H	L	P
P	H	E	P	A

Properties for above table operation \*

- C Y or N
- A Y or N
- Id Y or N
- Iv Y or N
- Cl Y or N

properties for # table

- C Y or N
- A Y or N
- Id Y or N
- Iv Y or N
- Cl Y or N

Key to Properties C = commutative , A = Associative,  
Id = Identity, Iv = Inverse, Cl = closure

A take home number sense problem which is optional, and will be given points based on how many turn in the Work sheet correct.

Show your work and be able to explain to the class how you arrived at the solution. I will randomly call on a few who got the correct answer to explain.

## Dogs

There were a total of four dogs staying at my parent's cabin this past Memorial Day. Aries, Liberty, Yankee and Murray are excellent dogs, so we frequently reward them with dog biscuits.

Saturday evening, I forgot to put away the box of dog-biscuits.

Aries found the box first and decided to eat three more than one-half of the biscuits. After Aries had finished snacking, Liberty found and ate three more than one-half of the remaining biscuits. When Liberty was finished eating Yankee found and ate three more than one-half of the remaining biscuits. Finally, Murray found and ate three more than one-half of the remaining biscuits.

When I awoke Sunday morning the entire box was empty. How many dog-biscuits were in the box originally?

Source: Greg Sarles, Bemidji State U.

Another short quiz based on your text assignments 1-5 will be given at the end of the next class period.

**SUMMARY LESSON 5:** This is meant as a brief introduction to some of the properties of whole numbers – a little if formalizing your mathematics that you will need for more advanced courses.

## Lesson 6

MN Standards: Develop meaning for integers and represent and compare quantities with them. Understand the meaning and effects of arithmetic operations with integers.

Since so many have trouble with Integers, specifically negative integers rather than just give out the rules we will investigate them in another and hopefully refreshing way.

1. Handout red and blue paper to everybody.
2. Get a partner
3. Fold and cut the papers into small squares or cubes  
The red squares each represent a Hot(H) unit or +1.  
The blue squares each represent a Cold (C) or -1.
4. Represent the number zero 3 different ways.
5. Represent the numbers 4, 6, -3, -5 each in three different ways.
6. Using Hot and Cold cubes show the operations  
 $4 + 2$ ,  $4 - 2$ ,  $-4 + -2$ ,  $2 + -4$ ,  $2 - 4$ ,  $4 + -2$   
Specify how the minus sign (-) is sometimes used as 'take-away' and sometimes referred to as Cold.
7. Using Hot and Cold cubes illustrate multiplication  
 $4 \times 2$ ,  $2 \times -4$ ,  $-4 \times 2$ ,  $-4 \times -2$ ,  $-3 \times -3$
8. Answer the questions do you see any patterns in multiplication that you discover?
9. Apply what we have learned in multiplication to division.  
 $-8 / 4 =$                        $-8 / -4 =$                        $8 / -4 =$   
Show how your answer is justified through multiplication.

Your text assignment will verify if you have learned how to use the operations on integers. Check your answers in the back of the text.

10. The last 15 minutes again will be a short quiz #2 from your text assignments.

**SUMMARY LESSON 6:** Hoping that the introduction of Hot and Cold cubes helped with your understanding of Integers.

## Lesson 7

1. Hand back and answer questions from Quiz #2.
2. Summarize and review Unit 1 -- the remainder of the class will be in answering questions on this unit, and preparing for the Unit Test the next Class period. The test will be 100 points and no calculator will be available. If it is necessary to use a multiplication table, a penalty will be assessed. Those penalty points can be regained by weaning yourself off those tables by the end of the semester. Also, because this is a major area of your basic skills and if you scored less than 70 points on the test, you need to get some help on what you missed and take a test on only what you were weak on to gain back up to 30 points. You will have to tell me in advanced when you are ready for this test, so I can put it up in the Testing Center for you to take at your convenience. You will need a picture ID to take it there.
3. When I return you test the next period after we have taken it, I will survey the class to here your opinion as to how you liked the method of teaching by using number sense ideas and investigations rather than just lectures and more traditional methods. So I need you to be thinking about that part too.