

# Assess Instructional Changes: Bias & Variability

## 1) Lesson Plans- what are you attempting to change or improve?

We are attempting to change the way kids learn data analysis and statistics, in particular bias and variability, by actively involving them in investigations.

## 2) What actual changes are you making?

In a traditional format, students would study bias and variability possibly by reading the terms and then calculating mean and standard deviation from a number of different example data sets to see how they relate to bias and variability. They would do a lot of the problems by hand, which could be quite cumbersome with the standard deviation formula. A traditional assessment would involve a test/quiz at the end with homework problems at the end. The lesson we are doing would let kids investigate a data set before getting in depth with statistical terms and let them actively study what bias and variability mean and how it connects to real world situations. For assessment, we would do a pre-test/post-test to see how much learning took place.

## 3) What effect should these changes have?

The changes from a traditional to an investigative approach would ideally have a positive effect on student learning. It also provides a group format that would help students learn different group dynamics. We hope these changes would show a statistically significant improvement in the way our students learn statistics.

## 4) Formulate hypotheses- null and alternative

Null Hypothesis: Change in instruction did not increase student knowledge.

Alternative Hypothesis: Change in instruction did increase student knowledge.

## 5) Experimental design for collecting data:

We would follow a Pre-Test/Post-Test assessment strategy where the students would take the exact same test before the unit on bias and variability and after the unit. Using a paired-data t-test from the scores they received on the two tests, we could conclude if the change in instruction increased the student's knowledge.

## 6) Data is collected, reviewed for problems and documented:

## 7) Data analysis- statistical tools you will use to analyze your data:

- i. Graphical tools: Bar graph, histogram, box plot, time plot,...
- ii. Statistical tools: mean, median, paired-data t-test, two-sample t-test,...

## 8) Statistical results and statements of conclusions:

## 9) Interpretation in the appropriate context

## 10) Action and dissemination

- i. Local – students, administrators, parents, community
- ii. State – conferences
- iii. National – conferences and journals

# The Dartboard Problem – Bias & Variability

**Activity Overview:** To introduce students to the concepts of bias and variability, the class will do an activity using a felt dartboard and Velcro balls (or some other type of safe and feasible dartboard and darts package). The class will be divided into groups and each group will collect a data set by throwing actual darts/balls at a dartboard. We will analyze the data the students collect both in groups and as a class. In addition to the initial activity, the students will measure how far off their throws were from the bull's eye to introduce the concepts of bias and variability. They will also use statistical software to calculate the mean and standard deviation of the errors of their throws. The final connection will be how the mean is the measure of bias and standard deviation is the measure of variability. Assessment will be based mainly on a Pre-Test/Post-Test but also on classroom discussions, group interaction, and group presentation of data.

## **Standards:**

The activities and discussions in this lesson address the following NCTM standards:

### *Measurement*

Apply appropriate techniques, tools, and formulas to determine measurements

- analyze precision, accuracy, and approximate error in measurement situations

### *Data Analysis and Probability*

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

- compute basic statistics and understand the distinction between a statistic and a parameter

Select and use appropriate statistical methods to analyze data

- for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics

## **Key Terms:**

Bias (accuracy)

Variability (precision)

Mean

Standard Deviation

## **Materials Needed**

String

Statistical Software (Fathom, Tinkerplots, Excel, TI graphing calculator)

Dartboards

Darts/Velcro Balls

## **Lesson Plan:**

~ 1-2 days: Introduction to Bias and Variability

**Pre-Test**

**Activity 1**

- 1) Break students into groups of about four depending on the size of the class
- 2) Assign each group darts and a dartboard and have each student in the group throw five darts/balls at the dartboard
- 3) Each group should record their throws on a “target sheet” that represents where their actual throws hit the dartboard

~ 1 day: Analyzing the Data

- 1) Start with a discussion on precision and accuracy
- 2) Each group will present their “target sheet” to the class by transferring their data onto a target on the board
- 3) The class will discuss each group’s dartboard and decide how they thought the group did in regards to accuracy and precision
- 4) Transition from the terms accuracy and precision to bias and variability with a classroom discussion

~ 1 day: Measures of Bias and Variability

- 1) Review the difference between bias and variability and what each relates to in regards to the dartboard (precision-variability and accuracy-bias)

**Activity 2**

- 2) Have students measure how far off each one of their throws are from the bull’s eye (their target) with a string/ruler
- 3) Each group should record their measurements and input them into some type of statistical software

~ 1-2 days: Analyzing Bias and Variability

- 1) Start with a discussion on what do we use to measure bias/accuracy and variability/precision
- 2) Have students calculate the mean and standard deviation of their data using some statistical software
- 3) Discuss how mean is the measure of bias and standard deviation is the measure of variability
- 4) Give the students some more example data sets where they can compute the mean and standard deviation (for example: **(Alternate Activity)** students can go search for data sets online and import them into a statistical software program and find the mean and standard deviation- another assessment option may be to have them present their data set to the rest of the class)
- 5) **Assessment on Bias and Variability: Post-Test**

# Assess Instructional Changes: Hypothesis Testing

## 11) Lesson Plans- what are you attempting to change or improve?

We are attempting to change the way students learn data analysis and statistics, in particular hypothesis testing, by actively involving them in an investigation.

## 12) What actual changes are you making?

We are changing from a lecture and practice format that is centered on students working on problems individually to an investigative format that emphasizes students working in groups to analyze data they generate on their own using various forms of technology to assist them in their learning.

## 13) What effect should these changes have?

These changes should lead to students having a deeper understanding of what hypothesis testing is and also enable them to learn how to design an experiment and analyze data they gather from an experiment. So the overall effect should be that they have a greater understanding of the topic presented.

## 14) Formulate hypotheses- null and alternative

Null Hypothesis: Change in instruction did not increase student knowledge.

Alternative Hypothesis: Change in instruction did increase student knowledge.

## 15) Experimental design for collecting data:

Our assessment would involve giving each student their own data set to analyze after the group projects were completed. They will have one day to complete a hypothesis test on their problem and then present their results to the class. This problem will be graded using a rubric from 1-5. All students will be given a copy of all the problems as a review.

## 16) Data is collected, reviewed for problems and documented:

## 17) Data analysis- statistical tools you will use to analyze your data:

- i. Graphical tools: Bar graph, histogram, box plot, time plot,...
- ii. Statistical tools: mean, median, paired-data t-test, two-sample t-test,...

## 18) Statistical results and statements of conclusions:

## 19) Interpretation in the appropriate context

## 20) Action and dissemination

- i. Local – students, administrators, parents, community
- ii. State – conferences
- iii. National – conferences and journals

# The Paper Airplane Problem – Hypothesis Testing

**Activity:** The class will design and fold paper airplanes to see who can come up with a plane that can consistently travel the longest distance. This investigation can be done with groups of up to 4 students or individually. This activity will describe how to perform this investigation with groups of 4 students. To perform this investigation talk about the type of experiment that the class will be designing and what question they will be trying to answer. Tell each group they have 5-10 minutes to discuss and design a paper airplane they think can travel the longest. Have each group decide on 4 roles for the experiment. The roles are as follows: a student who will throw the plane 30 times, a student who will measure the distances, a student who will confirm the measurements, and a student who will collect data. After data is collected all data will be entered into a classroom data table for all groups to record. After each group has collected their data and copied all other group's data they are to investigate the data set to see which group had the better design. At the end of the activity each group will have their conclusions about which group had the better design with evidence supporting their belief. This is a great opportunity for use of technology with fathom, tinkerplots, and Microsoft excel.

## **Standards:**

The activities and discussions in this lesson address the following NCTM standards:

### *Measurement*

Apply appropriate techniques, tools, and formulas to determine measurements

- analyze precision, accuracy, and approximate error in measurement situations

### *Data Analysis and Probability*

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

- understand histograms, parallel box plots, and scatter plots and use them to display data
- compute basic statistics and understand the distinction between a statistic and a parameter

Select and use appropriate statistical methods to analyze data

- for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics

## **Key Terms:**

Measures of center: mean, median, mode

Five Number Summary: Minimum, 1<sup>st</sup> quartile, 2<sup>nd</sup> quartile, 3<sup>rd</sup> quartile, and Max

Range and Interquartile Range

Box Plot

Histogram

Null and Alternative Hypothesis

Alpha value

ANOVA Test

p-value

### **Materials Needed:**

Paper

Graph Paper

String

Meter sticks and/or rulers

Fathom/Tinkerplots/Microsoft Excel

Graphing Calculator

### **Lesson Plan:**

~ 1 day: Introduction Data Collection

- 1) **Activity:** Divide the class into groups of at most four
- 2) Give each group 5-10 minutes to come up with a design for making a paper airplane
- 3) Have each group decide on a design they want to use and have them construct it using only one piece of computer paper
- 4) Each group will get 30 throws from a specific spot in the classroom to see how far they can get the airplane to fly
- 5) Each throw should be measured using string to reach from the throwing point to where the plane lands
- 6) Each throw should be recorded by a group member

~ 1 day: Continuing to Gather Data

- 1) Each group that wasn't able to finish the activity from the first day should throw, measure, and record their data
- 2) Once all the throws have been completed, the students should enter each group's data into some statistical software (i.e. TI graphing calculator, Excel, Fathom, or Tinkerplots)
- 3) Discuss with the class what they think is the best way to visually represent everyone's data (boxplot, histogram, dot plot, etc.)

~ 1 day: Analyzing the Data

- 1) Start the day out with a discussion on how the students will figure out which group had the best paper airplane (how is one group better than another?) Other questions you could asked: How do you answer questions like who has the best product, who had the best career in a particular sport, whose method or design is the best, or did a particular factor have some kind of influence?
- 2) Have each group compute the descriptive statistics for their data (their set of 15 throws)
- 3) Suggest that they organize their data into a box plot (hopefully suggested from the day before by a student) and discuss how to find the five number summary
- 4) The previous two steps should be done using some type of statistical software (Fathom, Tinkerplots, Excel, TI graphing calculator)
- 5) Have each group record data into a Class Data table so that each group will then be able to compare their group to the rest of the groups in the class.

~ 2-3 days: Analysis between groups: Hypothesis Testing

- 1) Start out this day by having each group decide by comparing class data who they believe had the best paper airplane design. After 10-15 minutes to present their statistics and evidence of who they believed had the best design. Have them provide a ranking of the groups in the class data table.
- 2) Introduce how you can statistically determine a difference between groups. Start by comparing 2 groups and talking about Hypothesis Testing. The students should be able to formulate the Null and Alternative Hypothesis by asking them what the purpose of the experiment was initially. What we will be testing is whether or not there was a statistically significant difference between the means of the 2 groups. The class will then learn how to use a T-test to test for a difference in the means. Formulate the null hypothesis that the means of the 2 groups are not different and have the student discuss in groups about the possible alternatives or outcomes. Then formulate an appropriate alternative hypothesis: one-tailed or two-tailed.
- 3) At this point you will need to cover with the groups how to set up a formal Hypothesis test. Below is the suggested format for hypothesis testing:

Hypothesis Testing:

- a) State the Null Hypothesis  $H_0$  and Alternative Hypothesis  $H_A$ . (This should be done before the data is collected and analyzed.
  - b) State the level of significant. How much error are you comfortable with. This is called the alpha value and most of the time 5% is used for the level of significance which is a probability of 0.05.
  - c) Collect data – Experimental, Survey, etc.
  - d) Analyze data with appropriate test
  - e) Make decision based on p-value: data supports  $H_0$  or data supports  $H_A$   
p-value  $> 0.05$  : fail to reject the null hypothesis  
p-value  $< 0.05$  : we reject the null hypothesis in favor of the alternative
  - f) Interpret Results
  - g) Action
- 4) Using some type of technology or software having students compare 2 groups that the whole class chooses. You will at this point have to guide the students through the use of technology. However have the students determine the null and alternative hypothesis within groups and enter data into software. Make sure they state the level of significance to be tested. You can choose talk about the types of tests usually used here. We will use a 2-tailed 2-Sample T-Test. Be ready to talk about when you would use a paired 2-Sample T-Test.
  - 5) Run test in software, analyze and interpret p-value. Either reject the null and accept the alternative or accept and go on believing the null hypothesis.
  - 6) Have students then compare all of the groups to each other and coming up with another ranking of which airplane design was the best.
  - 7) After performing test have students practice on other data sets paired and unpaired and one-tailed and two-tailed.
- \*\*\*Optional:**
- 8) Ask the class to comment on the possibility of comparing all groups at once: This can lead in to an ANOVA test which will compare groups to determine if there was at least one difference. You can use further test to determine which ones were different and how they were different.

## The Paper Airplane Problem

Group Members: \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Designing the Best Paper Airplane: You and your group are among many groups at a large corporation that will design paper airplanes for production next year. Teamwork plays a huge part in designing and a lot of times multiple teams are given the task of coming up with the new and latest design that will be manufactured. Before a design is accepted multiple statistical tests are performed to find the best design to be manufactured. It is that job of the class to find the team that can come up with an optimal design for a paper airplane. We have decided after designing our airplanes to collect data by testing the paper airplane in flight and recording the distances traveled. We hope to produce an airplane that will consistently travel the furthest distance. We will need to collect data, compute statistics, and compare the team's designs and then decide on the paper airplane we plan to send to the production line.

1) – Design a paper airplane

Discuss how your group decided on an airplane design that you felt would produce the longest flight and how include instructions on how you folded your airplane?

2) – Discuss the experimental design

How many throws are acceptable? Should more than one person throw the airplane? What other variables will affect the experiment between groups? Choose roles for the experiment: paper airplane thrower, recorder, person making measurements, and person reading measurements.

3) – Perform experiment and collect data

Number of Trials = \_\_\_\_\_

	Distance (cm)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

	Distance (cm)
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

	Distance (cm)
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	

4) – Analyzing the Classroom Data:  
Classroom Data Table:

Group #	Distance (cm)	Group #	Distance (cm)	Group #	Distance (cm)	Group #	Distance (cm)	Group #	Distance (cm)
1		1		1		1		1	
2		2		2		2		2	
3		3		3		3		3	
4		4		4		4		4	
5		5		5		5		5	
6		6		6		6		6	
7		7		7		7		7	
8		8		8		8		8	
9		9		9		9		9	
10		10		10		10		10	
11		11		11		11		11	
12		12		12		12		12	
13		13		13		13		13	
14		14		14		14		14	
15		15		15		15		15	
16		16		16		16		16	
17		17		17		17		17	
18		18		18		18		18	
19		19		19		19		19	
20		20		20		20		20	
21		21		21		21		21	
22		22		22		22		22	
23		23		23		23		23	
24		24		24		24		24	
25		25		25		25		25	
26		26		26		26		26	
27		27		27		27		27	
28		28		28		28		28	
29		29		29		29		29	

30		30		30		30		30	
----	--	----	--	----	--	----	--	----	--

5) Data Analysis: Using technology and/or computer software and the space below compute summary statistics and graphs for each group that your group believes will help you in your decision to pick which airplane design is best. You need to record statistics computed and examples of graphs in the space below.

6) Provide a ranking from the best design to the worst design and explain using statistics why they are ranked in this order.

7) Conduct a hypothesis:  
Hypothesis Testing:  
 $H_0$ :  
 $H_A$ :  
Level of significance:  
  
Type of test performed:  
  
p-value:  
  
Conclusion:

- 8) In your groups compare all of the groups using the appropriate hypothesis test and rank the airplanes from best to worst with your results. Does this agree with your previous ranking or did your order change at all. Explain why this might of happened.

Optional Item:

- 9) When we compare more than 2 groups using 2-sample T-Tests the amount of error in our decision is increased. A better test for comparing means within 3 or more groups is an ANOVA test. Perform a hypothesis test using an ANOVA test, which will test for at least one difference in the means within the design groups.

Hypothesis Testing:

$H_0$ :

$H_A$ :

Level of significance:

Type of test performed:

p-value:

Conclusion:

- 10) Action – As a class decide which design was best and that the class would send to the production line and why based on the classroom statistics