Mathematical Card Tricks

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I. Introduction

When thinking of card tricks, images of cards disappearing under tables and sleight of hand techniques generally come to mind. Contrary to these perceptions, many tricks do not fit those descriptions, and their foundations are strictly mathematical, some at an algebraic level and others using higher mathematics. By exploring the mathematical foundation behind how a trick works, the trick can become much more enjoyable and meaningful to the common observer.

Before exploring card tricks and their mathematical foundations, it is necessary to have an understanding of the history of cards as well as the significant role shuffling techniques play in the performance of card tricks. A collection of known card tricks will be explored, along with independent discoveries of unknown mathematical principles. Finally, a discussion will be held on how card tricks can be effectively used in an educational setting.

II. History

It is thought that Arabs brought cards from the Middle East in the fourteenth century, and they were the ones to introduce them to Europe. Though some discrepancies between sources do occur, the common agreement is that the four suits symbolize the four seasons, the thirteen cards represent the thirteen lunar cycles each year, the twelve court cards correspond to the twelve months of the Gregorian calendar, and the fifty-two cards are the fifty two weeks in the year (Ochab).

The notion of card tricks began soon after cards were introduced, with the first known reference dating back to 1408. It is conjectured that gamblers in those days wanted to increase their chances of winning and thus developed the first card trick techniques. The first card effect to be described and explained in print appeared in 1550 in Girolomo Cardano's *De subtilitate*.

This effect was the location identification of a selected card. Three methods were mentioned in which mathematical methods were used. Later, an expanded edition of this work was published and Cardano added more tricks. Cardano also published *Ludo aleae*, which was seen by others to be a gambler's manual. Cardano was a gambler, and this work contained the odds he created for events such as throwing dice and playing with cards. He dealt with probabilities of some of the simple draws in card games. These probabilities are what led to the beginning of fascination with card tricks (Cardano).

In 1592 Horatio Galasso published *Giochi di carte bellissimi diregola, e di memoria.* In this work Galasso described tricks that involved mathematical principles rather than the sleight of hand techniques. Since then, cards have been a fascinating topic because of the symbolism, numerical properties, and mathematical ideas embedded within them (Ochab).

III. Card Shuffling

The act of shuffling itself is an extremely important aspect of card tricks. Though it may seem like an unlikely aspect of the trick itself, often the shuffling techniques are the most important part. Martin Gardner has done much work in the area of shuffling and the various shuffling techniques that can be used. Some specific types of shuffles, which he describes in his book Mathematical Carnival, are known as overhand, riffle, in-shuffle, and out-shuffle.

In the overhand shuffle, the cards are held by their ends in the performer's right hand, and the left thumb removes the cards off the top in small, random size packets. A perfect overhand shuffle occurs when the thumb takes one card at a time and does not destroy the order of the deck at all. It simply reverses the original order, and thus a second perfect overhand shuffle will restore the original order (Mathematical Carnival 124).

The perfect riffle shuffle, which is known to American magicians as the faro shuffle and to English magicians as the weave shuffle, is one in which the cards drop one at a time, and alternately from the two thumbs. If it contains an even number of cards, it must be divided exactly in half before the shuffle begins. If the deck contains an odd number of cards, the deck should be divided close to half as the smaller half shuffles into the larger one so that the top and bottom cards of the larger half become the top and bottom cards of the deck after the riffle shuffle is completed (Mathematical Carnival 125).

Variations of the riffle shuffle are the out-shuffle and in-shuffle, and these also play a role in the life of a card magician. If cards previously at the top and bottom remain at the top and bottom of the deck, it is known as an out-shuffle. If the first card to fall is from what was formerly the top half of the deck, and the former top and bottom cards go into the deck to positions second from top and bottom, it is known as an in-shuffle. A deck of n cards, given a repeated series of riffle shuffles of the same type, will return to its original order after a finite number of shuffles. If n is odd, the deck returns to its initial state after x shuffles, where x is the exponent of 2 in the formula $2^x = 1$ (modulo n). If the deck is even, the number of out-shuffles needed to restore the original order is $2^x = 1$ (modulo (n – 1)). The number of in-shuffles needed to restore the original order is $2^x = 1$ (modulo (n + 1)) (Mathematical Carnival 125-127).

One of the most surprising theorems in card shuffling is known as the Gilbreath principle and can be viewed as a card trick in itself. For this trick, arrange a deck so that the colors alternate. Then cut it so that the bottom cards of each half are different colors, and then riffle shuffle the halves together. Then take cards from the top in pairs, and you will find that every pair will have both a red and a black card (Modeling 173).

A short proof by induction can show that this must happen every time. Assume that the first card to fall on the table during the shuffle is black. If the next card to fall is the card directly above it in the same half, that card will be red, thus creating a red-black pair. If the next card to fall is in the other half, it will also be red, thus still creating a red-black pair. In either case, after two cards have dropped, the bottom cards of each half will be of different colors, so the situation is exactly the same as before. The same argument applies for the rest of the cards, so no matter how carefully the deck is shuffled, it will create a pile of red-black pairs (Modeling 173).

IV. Exploration of Known Card Tricks

There are many card tricks for which the mathematics behind them has already been discovered or that were designed with the mathematics in mind before all details of the trick were designed. To begin, we can look at a simple trick entitled *Number's Game*. For this trick, the deck of cards consists of values 2 through 9 and the Aces, with aces having the value of 1 and all remaining cards have their corresponding numerical value. To perform the trick, a person picks a card out of your deck, without showing it to you. Then they take the value of that number and multiply it by two to obtain a new number. To that new number they add five and then multiply the most current result by 5. The player now proceeds to choose another card. They then add the value of that card to the previous value they obtained and then tell you the final number the obtained (Card Trick Central). After these steps, anyone can be just a simple algebraic equation away from appearing to be an astonishing magician! The explanation behind the trick is as follows:

Let n be the first card you chose. Then we multiply that by 2, giving us 2n. Next 5 is added to the result, yielding 2n + 5, and then the entire quantity is multiplied by 5, leaving us with 5(2n + 5). Let the second card chosen be the value m, which is then added to the previous result. So we now have 5(2n + 5) + m, which is equivalent to 10n + 25 + m. If 25 is subtracted from this equation, we have 10n + m, which can easily be recognized as the expanded form of the two digit number nm. Therefore, the two cards they have picked are the two digits of the number obtained after subtracting 25 from the original number that you are told (Card Trick Central).

Another algebra trick to explore has no specific name, but will be referred to as *Bottom's Up*. The ultimate goal in this trick is for the performer to determine the sum of the bottom cards in each stack that is made. For this trick, a standard deck of 52 cards is used. The numerical value of the cards are as follows: each ace counts as 1, twos through tens count as their face value, and each jack, queen and king count as 10. Out of the performers sight, an assistant forms separate piles of cards by first noting the numerical value of the first card picked and then placing it face down. Now additional cards are counted from the deck until the count reaches 12. For example, if the first card was a nine, the dealer would silently add three more cards to the deck by counting "ten", "eleven", and "twelve." This process of making stacks that add up to 12 continues until either all of the cards are used up or there are not enough cards to complete a stack. The person who has just finished dealing the cards will then proceed to tell you how many stacks were created and how many leftover cards there were. From just these two pieces of information, the sum of the bottom cards on each deck can be found without knowing the value of any card (Lindstrom). The solution to this trick can be found using counting techniques and algebra.

The mathematical explanation of *Bottom's Up* requires two principles which are fairly self-explanatory, yet are important to be aware of. The first principle is that the sum of the number of cards in different piles equals the total number of cards in the deck. The second principle to recognize is that if a pile is formed with a card whose bottom value is B, and whose top card is assigned the value of T, then the number of cards in the pile is (T + 1) - B.

Now, if we assume there are k piles formed, then we can

let B_1 = the numerical value of the bottom card of the first pile

B_2 = the numerical value of the bottom card of the second pile

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.
.

B_k = the numerical value of the bottom card of the k^{th} pile.

Then by the second basic principle, there are

$(12 + 1) - B_1 = (13 - B_1)$ cards in the first pile,

$(13 - B_2)$ cards in the second pile,

.
.
.

$(13 - B_k)$ cards in the k^{th} pile.

We can also designate D as the number of cards that are left over after the k piles have been formed. Then by the first basic principle, we know the following equation:

$$(13 - B_1) + (13 - B_2) + \ldots + (13 - B_k) + D = 52$$

Since we are trying to determine the sum of the bottom cards of the stacks, we know that we want to solve for $(B_1 + B_2 + \ldots + B_k)$. By using algebraic properties, we can manipulate the above equation in the following manner:

$$(13 - B_1) + (13 - B_2) + \ldots + (13 - B_k) + D = 52$$

$$(13 + 13 + \ldots + 13) - B_1 - B_2 - \ldots - B_k + D = 52$$

$$13k - (B_1 + B_2 + \ldots + B_k) + D = 52$$

$$13k + D - 52 = (B_1 + B_2 + \ldots + B_k)$$

From the equation derived above, we can clearly see that from the stacks that we created, in order to determine the sum of the bottom cards, we need to multiply 13 by the number of piles that were created, subtract 52 from that value and then add the number of cards that were leftover and not included in any stack (Lindstrom). This process will work every time and will amaze your audience without fail.

Cyclic properties can also be seen in card tricks, as is the case in *Unfortunate 57.* While the performer shuffles the deck, the first spectator should be asked to name their favorite suit. Then another spectator must riffle shuffle the deck twice. Meanwhile, the third spectator will pick a number that is between 1 and 7. Then the performer takes the deck and removes the first six single digit card values that are found in the suit named by the first spectator, and then write down the number formed by these digits. The third spectator then multiplies this number by the number between 1 and 7 that they picked, and that number should match the number written on the prediction slip (Mulcahy).

The method and mathematics of *Unfortunate 57* is based on the cyclic property of the number 142857. This number is the repeating cycle part of the decimal representation of 1/7. If you multiply 142857 by any number from 1 to 6, you get back the same six digits but they will be permuted cyclically. Since it is always easy to determine the last digit in each case, the cyclic property allows the performer to predict the entire answer with very little effort (Mulcahy). Any other numbers that exist with this same cyclic property could be used in a similar fashion.

The concept of permutations in Modern Algebra can also be used to create some card tricks. Bob Hummer was one of the earliest innovators in this field and his trick is known as *Hummer's One-Two-Three Trick.* In this trick, any Ace, 2, and 3 are placed face up in a row on the table. Then a spectator is to mentally choose one of the cards, turn it over on the table, and then turn over the other two cards, but switch their positions first. Then the performer collects the cards and cuts them several times. Following this, the cards are placed face down in a row on the table again. The spectator then guesses the location of the card they mentally chose, and turns that card over, but does not let the performer know if their guess was correct. The performer can then instantly tell whether they are correct or not, and if incorrect, the performer will point out the correct card (Mulcahy).

The method for this trick begins with picking up the cards so that from top to bottom you have the rightmost card, the middle card, and the leftmost card. Then cut the cards to the bottom, one or two at a time. Do this repeatedly while secretly keeping count, and stop when 10, 13, or 16 cards have been moved. Then deal out the top card to the middle, the second card to the right, and the third to the left. Adjust your mental picture of these three positions to read 3, 2, 1 from left to right. Then the spectator is invited to turn over whichever card they think is theirs, call it position i. If the card with value i is in this position, then the spectator has chosen the correct card. If not, then the card with value j not equal to i is in position i, and the performer can quickly turn over the card in the third position k not equal to i or j and correct their choice.

The mathematics behind *Hummer's One-Two-Three Trick* has to deal with permutations in the symmetric group on {1, 2, 3}, which we take to represent the initial positions of the Ace, 2, and 3. The spectators actions then affect an unknown transposition k. The way that the cards are picked up reverses their order, which is then equivalent to the permutation (1 3). When s cards are cut to the bottom of the pile such that $s = 1 (\mathrm{mod}\ 3)$, the pack become the 3-cycle (1 3 2). This makes the final placement of the cards the same as the transposition (1 2). Since $(1\ 2)(1\ 3\ 2)(1\ 3)k = k$, the result is that knowing just one of the cards tells the performer what the identity of k is, but the switching makes it seem less obvious to the spectator and makes it seem even more like magic. This trick could be extended to look at other symmetric groups and other pre and post multiplications. What happens if the symmetric group is larger? If we look at a different symmetric group, can we simply use different groups to pre- and post-multiply by to obtain a similar result?

Number theory is also an aspect of mathematics that contains some very worthwhile mathematical card tricks that can be explored. The number theoretic trick to be described is known as *The Most Complicated and Fantastic Card Trick Ever* and was investigated by Kurt Eisemann. This trick was developed by Martin Gardner, who also coined the name. There are two decks in this trick, a black deck consisting of the spades from Ace through Queen, and a red deck consisting of the hearts from Ace through King. These decks will be sequenced in a particular manner. Then the red deck will be cut any number of times and an arbitrary number can be chosen by an audience member to determine the number of cuts. That arbitrary number will be the number of piles dealt from left to right to form what will be known as heaps. Then the heaps will be consolidated into one single deck, and then the red deck will be cut one final time. To complete the reorganization process of the decks, the black deck will also be cut in a certain manner to create a new sequence of black cards. Once this process is completed and the cards are dealt into two rows, one row consisting of the black cards and one row consisting of the red cards. From these rows, if the performer chooses one card from the black deck, the performer will be able to pinpoint the location of one specific card from the red deck, and vise versa.

The mathematical explanation of *The Most Complicated and Fantastic Card Trick Ever* begins by sequencing the black deck. The spades are ordered so that the successive cards represent the successive power residues modulo 13 of its primitive root 2. Thus the ordering of cards is 2, 4, 8, 3, 6, 12, 11, 9, 5, 10, 7, and 1. This deck can be cut any number of times before beginning because any cyclic permutation of the cards does not affect the properties that will hold throughout the trick. Next, we need to sequence the red deck. This is done by creating a property of reciprocity that will occur between the red and black decks, meaning that if the face value n appears in position number i, so that $b_i = n$, then the red deck correspondingly must show in position n the face value i, so that $r_n = i$. For example, if the 2 of spades is in the ninth position of the black deck, place the 9 of hearts in the second position of the red deck.

Before we continue, definition of a few variables is necessary. Let f represent the face value of the first card immediately following the Ace of spades, let e represent the number of cards lying at the end of the black deck beyond the Ace of spades, and let x(n) represent the index of n mod 13. By construction of the black deck, position i is occupied by a spade with face value of

$$b_i = f^{\,s} (\text{mod } 13), \text{ where } s = i + e \ (\text{mod } 12)$$

In other words, the face value $n = b_i$ is found in position number $i \equiv x(n) - e$. By construction of the red deck, position n is occupied by a heart of face value $r_n = i \equiv x(n) - e$ (mod 12).

Next we must go through the shuffling process of the red deck. First we create the k heaps based on the arbitrary number chosen. During the process the last card will fall on a certain heap, and we will let that heap be denoted z, where $p = mk + z$, $m \in \mathbf{Z}$, and $(k, z) = 1$ because any common factor would also divide p. Hence, none of the cyclic counts of k position numbers will end on a vacant position until all heaps have been used. Heap number i has as its bottom card the face value i, and its top card $mk + i$ when $i \leq z$ or $(m - 1)k + i$ when $i > z$.

Within each heap, successive cards have the common difference k. When heap i is placed on top of heap $i + z(\text{mod } k)$, the difference d between the bottom card and the top card are both congruent to k(mod p). The formula given for reassembly of heaps tells us that the effect of the k-shuffle is to increase (mod 12) the face value of each card of the red deck by the same constant. At the beginning, $r_n = i$ corresponded to $b_i = n$. For the new sequence R_i, the original black deck is updated by moving $(R_i + e)(\text{mod } 12)$ cards from the bottom of the black deck to the top. Thus, even though the order of the cards has been drastically changed, the reciprocity property still holds.

Many extensions to the most complicated trick can be explored. One could look at various permutations to determine if the same process would work with other primitive roots. Also, the shuffling procedure was applied to the black deck instead of the red deck, so we could change that process to see what effect it would have on our final product. Finally, we could also investigate what would happen if the number of cards in the red deck was not prime.

V. Independent Explorations

From the book Self-Working Card Tricks by Karl Fulves, I have independently explored the mathematical foundations behind two tricks. The first trick is entitled *The Magic Thirteen*. The goal of this trick is for the dealer to tell the two participants which cards they have chosen. First 13 cards are dealt from the top of any shuffled deck. One of the participants gives an arbitrary number of the cards to the second participant and keeps the remaining cards. Each participant then is instructed to remember the number of cards in their hand. The cards are then dealt face up off the top of the deck. Each person will remember the face value of the card that corresponds to the number they are remembering. For example, if a participant had five cards in their hand, they would remember the 5^{th} card that is dealt off the top of the deck. Two piles will then be created. When one participant tells that they have seen their card, the performer will know which card the second participant has chosen (Fulves 22-23).

The key to performing this trick successfully is when the cards are distributed into two decks. The cards are first dealt to the left deck, then to the right, and alternating thereafter until all the cards are distributed. Consider each of the cards as being labeled $C_1, C_2, \ldots, C_{13}$. Then the order of the deck is as follows:

C_1		(bottom)
C_3	C_2	
C_5	C_4	
C_7	C_6	
C_9	C_8	
C_{11}	C_{10}	
C_{13}	C_{12}	(top)

Now we will flip the left had heap face up, changing our order to the following:

C_{13}		(bottom)
C_{11}	C_2	
C_9	C_4	
C_7	C_6	
C_5	C_8	
C_3	C_{10}	
C_1	C_{12}	(top)

Now by starting at the top of the deck and taking a card off of each pile, the corresponding cards are the two that are remembered by the participants because they are such that the indices of each corresponding pair adds to 13. Thus, once one participant tells the performer that their card has been found, you will automatically know that the other participants card corresponds to it.

Though this trick specifically uses 13 cards, our trick can be expanded to use any odd number and still will produce the same result. This can help add some mystery and magic to your audience by using nine cards in one instance and then fifteen in the next.

The next trick explored from Fulves' book is known as *A Card and a Number*. In this trick, the goal is to determine the card that the participant has chosen after moving the cards around in the deck. First the participant chooses a number between 1 and 10. The dealer then removes that number of cards from the deck. The participant then will take note of the card in the deck which corresponds to the number of cards removed. For example if there are six cards removed, the participant will remember the sixth card in the new deck. The participant then thinks of another number between 25 and 40. Without knowing the card that the participant is remembering, the performer will make that card be in the position in the deck the same number down as the second number that was chosen by the spectator (Fulves 74-75).

To explain this trick, let k be the number of cards removed from the original deck. Then our new deck has 52 – k cards, and the participant is remembering the value of the k^{th} card. Now let n be the number chosen by the second spectator. Start with the number n + 1 and move the cards to the bottom of the deck, counting until you reach 52. This means 52 – n cards have been moved to the bottom of the deck. Subtracting cards at the bottom of the deck from cards at the top do the deck, we see that there will be $(52 - k) - (52 - n) = -k + n$ cards at the top of the deck. Now if we count down n cards in the top deck, we are at the (-k) card in the deck. However, the (-k) card in the top deck is the same as being at card k in the bottom deck. Since card k is the card the participant is remembering, the card we are looking for has now been found.

VI. Card Tricks in an Educational Setting

Children of all ages are fascinated by magic tricks, and mathematical magic tricks are no exception to that. For that reason, incorporating card tricks in the math classroom can prove to be a very useful way to introduce topics and can also be a great way to promote interest in the subject. In looking for the mathematics involved in a trick, students would have to engage in several important mathematical tasks, such as problem solving, making conjectures, testing conjectures, and verbalizing their results. Going through this process would be an important for anyone in the learning process and would ensure that true learning would actually occur.

In Principles and Standards for School Mathematics from the National Council of Teachers of Mathematics, 10 standards were established. In addition to the five content standards, the other five are problem solving, reasoning and proof, communication, connections, and representation (Principles). All five of these standards are incorporated into the solving of card tricks, thus reaffirming the belief that card tricks are worthwhile mathematical activities.

One such trick that would be very useful in a classroom setting is unnamed, but was discussed by Catherine Mulligan in an issue of Mathematics Teacher (Mulligan 100-103). It involves the person performing the trick to prevent three chosen cards from being dealt to the player, even though the person does not know what the cards are. To start, the player draws three cards from the deck, making sure to not show them to anyone. The remaining cards are then separated into piles of 5, 15, and 15, while the remaining 14 are put aside. The player must remember the number and suit of all three cards chosen, and then must place the first of the cards on top of the pile of 5. The player will then move some of the cards from the first pile of 15 and place them on top of the pile that now has 6 cards. The second card will then be placed on top of the remaining cards in the first pile of 15, and just like before the second card will be covered with some arbitrary number of cards from the second pile of 15. Now place the third card on top of what is left from the second pile of 15. Finally, place the last pile of 14 cards on top of the pile that was originally the second pile of 15. All the cards must be face down at this point. Now all three piles are combined by first picking up the pile containing the third chosen card, placing this pile on top of the pile containing the second chosen card, and then placing all these on top of the pile containing the first chosen card.

Now the cards must be dealt from the top of the deck into two piles dealing first to the person performing the trick and then to the player and alternating until all the cards are dealt. Then after determining if the player has any of the original three cards, the performer again divides up the cards in their pile between the two people in the same manner as before. The third time the performer will again split up the cards, this time first dealing to the player and then alternating. Again, after determining if any of the three cards originally noted by the player are in the players pile, the performer will deal the cards one more time, this time giving the first card to the performer and then alternating. There should now be three cards left in each person's hand. The remaining three cards in the dealer's pile are the three cards originally chosen by the spectator (Mulligan 100).

The key to this trick can be realized using either algebra or arithmetic, which is what makes it such an amazing addition to almost any classroom. To simplify our notation, let F be the first chosen card, S the second chosen card, and T the third chosen card. When students begin by analyzing the positions of the cards within the deck, they should realize that the key idea is that the cards F, S, and T are all in odd positions from the top of the deck. This means the player would not receive any of those cards on the first deal, and the same is true for all remaining deals. This results in the dealer successfully completing the trick by finding all three of the originally chosen cards (Mulligan 101).

More specific algebraic notation can also be used to explain this trick. For this we can let n be the bottom relative position of one of the three chosen cards. That makes its position relative to the top $53 - n$. Then the first card is dealt to the dealer, so its position from the bottom is $(53 - n + 1)/2$ and its position relative to the top is $27 - (53 - n + 1)/2 = (n/2)$. The second deal gives again to the dealer first in position $((n/2) + 1)/2 = (n/4) + (1/2)$ from the bottom or 14 -

(n/4) + (1/2) from the top. Deal three is when the dealer deals first to the player, and this places the cards in position (14 - (n/4) + (1/2))/2 = 7 – (n/8) – (1/4) from the bottom or (n/8) + (1/4) from the top. On the last deal, the card is in position (n/16) + (5/8) from the bottom or (27/8) – (n/16) from the top. In each case the solutions will always be that the original bottom relative positions of the three cards must be n = 6, 22, and 38, and these positions will always be the last three in the spectators hand, thus creating a trick that works every time (Mulligan 102-103).

This trick can incorporate several great pieces of mathematics. First students must find a way to represent the cards to determine a pattern. This will usually result in the use of tables by the students. The students might introduce symbolic notation, and if not, this would be a perfect opportunity to incorporate that idea. Other mathematical concepts can also be explored at this time. One such idea is an odd number minus an even number is always an odd number. Another is that the number of even numbers from 1 to any even number is always half of the highest even number that you count up to.

Another card trick exists that works well in a classroom and can be explained using first year algebra. To begin, shuffle the deck and place cards face up all in one stack. As the cards are being placed face up, secretly pick out and remember one card. Then continue dealing so that you place exactly ten more cards face up on top of the secretly selected card. Then have three students each select one random card. Have these three cards placed face up in three separate piles. Turn over the face up pile of cards containing the secretly selected card and place it under the stack of cards in your hand. You now have three cards, each facing up, and one stack of cards in your hand. Now each face up card is worked with separately. Place additional cards face down on top of each of the three face up cards. Start with the value of each face up card and add cards until you reach a count of thirteen. Let the bottom face up card stick out a little so you can use it in the next step. Now out loud add the values of the three face up cards on the bottom of the piles in view. Call this sum S. Ask if anyone knows the value of the S^{th} card in your hand. Pretend you have to struggling to recall it, meanwhile, count out S cards and you will have found the card (Davis 326-327).

To determine how this trick works, first find an expression for the total number of cards in each of the three piles of cards left in view. If n is the value of the face up card on the bottom, there 14 – n cards in the pile. If n_1, n_2, and n_3 are the specific values of the face up cards on the bottom, then there are $(14 - n_1) + (14 - n_2) + (14 - n_3)$ cards on the table. In the last step of the trick, you counted to the $S = n_1 + n_2 + n_3$ card down in the cards in your hand. Adding the number of cards on the table and S gives $(14 - n_1) + (14 - n_2) + (14 - n_3) + (n_1 + n_2 + n_3) = 42$. You will always reach card number 42. There are 10 cards beneath the secretly selected card because they were put there in the second step. That means the secretly selected and memorized card is also the number 42 (Davis 327).

For any trick shown to students, there are many helpful questions and suggestions that can be used to help students find the algebraic explanation. One is to ask them to look at the total number of cards in each pile. Ask them how they can predict each total if they know the value of the bottom face up card. They can also think of the cards as all being in one stack instead of individual piles and then try to determine how far down the secret card is in the deck.

It is also important to have students look back at their work and reflect upon the conclusions they have reached. To guide the students in this process, ask if the trick will always work or if something will cause the trick to fail. Also ask the students if we counted to different numbers, what other changes or adjustments would have to be made. Also, why is the number of cards in each pile the way we have stated? What could happen if more students selected cards than the number originally chosen? What changes would you make in the trick if you had a double deck to work with? All of these are valid questions that will encourage deeper thinking on the part of the students, and will be a great source of enjoyment for them as well.

As with all card tricks, added theatrics can reinforce the positive effects in the classroom. Pretending to struggle to recall the memorized card and claiming to have memorized a long sequence of cards are two such methods. Holding off an explanation or limiting the number of performances in a day are other ways that could spark additional interest (Davis 328). Once students know how to do the trick, they are usually very interested in challenging themselves to figure out why it works, and the challenge makes it much more rewarding for the students when the result is finally determined.

VII. Conclusion

Though card tricks are fun and exciting to watch and participate in, the mathematics behind them lends to the understanding and appreciation at a whole new level. The mathematics behind card tricks can also help to spark the interest of many people who would usually not be excited about math and its applications, which can be a very positive result of the tricks. A variety of levels in mathematics can be found in card tricks so they can be used in almost any mathematical level to help create a fun way to look at some mathematical ideas. By exploring these ideas and analyzing their outcomes, we can see how important and applicable the mathematics we study can be.

Card tricks are always fun and have proved to be a wonderful means of interesting others, both mathematicians and non-mathematicians alike, in the subject of mathematics. Once a trick has been performed, the first question that comes to mind is "How did they do that?" Often people will begin the process of determining the secrets behind the trick. They will become 'sucked into' the mathematics after that point and there is no escape! These tricks appeal to both young and old, and are especially beneficial in an educational setting. Even the most reluctant student will be intrigued with the mathematics underlying the trick and possibly by the extent of hidden mathematics in real life which they may not even realize.

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