Matrices and the Game of Chutes and Ladders

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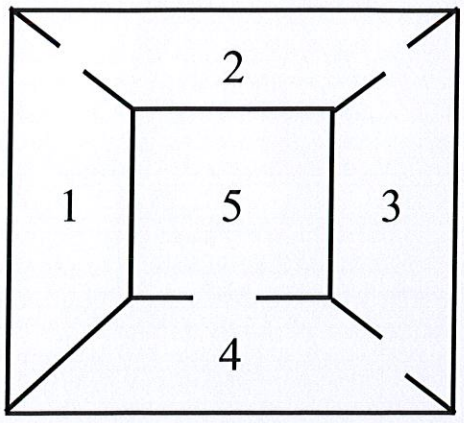
When I started this thesis, I wanted to work with regular Markov Chains and look more closely at the theorem I had begun working with for my Senior Research Project. A regular Markov Chain is a matrix in which all of the entries are positive for some power n. The theorem states that if P is the transition matrix for a regular Markov chain and w is the steady state vector, then

I) $w = Pw$,
II) $P^n \rightarrow W$ as $n \rightarrow \infty$, where each column of W is the same probability vector w which has positive components,
III) if p is any probability vector, $P^n p \rightarrow w$.

For my Senior Research Project I first showed an example in which this theorem holds true for a 2x2 matrix and then proved the theorem for a general 2x2 matrix. I was originally going to take this further by investigating the general 3x3 case for my thesis, but chose to move in another direction while still working with transition matrices. I loved covering matrices in Linear Algebra and continuing with something I enjoyed seemed like a good idea.

At this point, I believe it is important to give a brief example of what a transition matrix is and how it works. Let us say we have a mouse that is moving through a box like this one.



We can then make a transition matrix to show how the mouse will move through the box. If we say that the mouse is equally likely to stay in the room that he is in or move to another room, we have the following transition matrix.

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & 1/2 & 1/2 & 0 & 0 & 0 \\ 2 & 1/3 & 1/3 & 1/3 & 0 & 0 \\ 3 & 0 & 1/3 & 1/3 & 1/3 & 0 \\ 4 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 5 & 0 & 0 & 0 & 1/2 & 1/2 \end{array}$$

This transition matrix gives us the probabilities of the mouse being in any one room after only one move. For example, if the mouse is in room three, the probability that he will be in room four after one move is 1/3. If the mouse in is room one, the probability that he will be in room five after only one move is zero because it is impossible to get from room one to room five in only one move. If we calculate the second power of the transition matrix we will have the probabilities of the mouse being in any one room after two moves. This matrix is as follows:

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & .417 & .417 & .167 & 0 & 0 \\ 2 & .276 & .389 & .222 & .111 & 0 \\ 3 & .111 & .222 & .333 & .222 & .111 \\ 4 & 0 & .111 & .222 & .389 & .278 \\ 5 & 0 & 0 & .167 & .417 & .417 \end{array}$$

If we investigate this matrix, we see that the probability of the mouse moving from room three to room four after two moves is about 22 percent. But again, the probability of the mouse moving from room one to room five in two moves is zero, again because it is impossible for the mouse to get to room five in only two moves. If we calculate the third power of the transition matrix we have the following matrix:

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & .347 & .403 & .194 & .056 & 0 \\ 2 & .269 & .343 & .241 & .111 & .037 \\ 3 & .130 & .241 & .260 & .241 & .130 \\ 4 & .037 & .111 & .241 & .343 & .269 \\ 5 & 0 & .056 & .194 & .403 & .347 \end{array}$$

This matrix gives the probabilities that the mouse will be in any one room after three moves. Here again, we find that the mouse is still unable to get to room five after starting out in room one because this probability is still at zero. Next we calculate the fourth power of the transition matrix.

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & .308 & .373 & .218 & .083 & .019 \\ 2 & .248 & .329 & .231 & .136 & .056 \\ 3 & .145 & .231 & .247 & .231 & .145 \\ 4 & .056 & .136 & .231 & .329 & .248 \\ 5 & .019 & .083 & .218 & .373 & .308 \end{array}$$

Here for the first time we see that the mouse is able to move from room one to room five. Because it is the fourth power of the transition matrix that this first becomes possible, it will take the mouse a minimum of four moves to move from room one to room five. Again, this is just a brief introduction to transition matrices and will be referred to later in the paper.

I was introduced to the idea of analyzing Chutes and Ladders by Dr. Eric Lund. He had heard about it from Dr. Derek Webb, who had a professor in college investigate the game. It seemed like an interesting idea so I did some searching on the Internet and found a few sites in which different people had looked at the game of Chutes and Ladders. Analyzing the game involves looking at the minimum and maximum game lengths as well as finding an average game length. I talked the idea over with Dr. Lund and we decided it would be an interesting idea to pursue.

The game of Chutes and Ladders is one that has been around for a while. I remember playing it as a child and have played it with children I have babysat. It is a fun game and lends itself easily to mathematical analysis. The question arises, why analyze Chutes and Ladders? One reason is simply that it is a game I remember playing, and because it is fun to play it seems that it would also be interesting to analyze.

The game of Chutes and Ladders consists of a board labeled 1-100 and the object is to be the first to land on space 100 and thus be the winner. You move around the board by spinning a spinner numbered 1-6 and moving the number of spaces. If you land on a ladder you get to move up the ladder and thus advance further than your spin, but if you land on a chute you slide down and move back down the board (See Appendix 1, Pg. 19).

Analyzing Chutes and Ladders involved making a 101x101 transition matrix. The game board, as stated, consists of spaces 1-100 but a player starts off the board on space "0". Thus to accommodate this extra space, we use a 101x101 transition matrix. Taking powers of this matrix is what allows a minimum game length to be found which is one of the things I was interested in knowing. Analyzing the game also involved writing a program to simulate the game in order to find an average game length. Taking on a 101x101 matrix seemed a little daunting at the time, so I made a smaller version of the game. It is a board with spaces labeled 1-36 and only two chutes and two ladders (See Appendix 1, Pg. 20). Analyzing this game would only involve a 37x37 transition matrix and a much easier program to write.

The transition matrix for this game involved making a 37x37 matrix and then filling in the rows and columns with the correct probability. For example, if a person starts the game on space

"0" they have a 1/6 probability of landing on space one, a 1/6 probability of landing on space two, etc., up to space six. So in the matrix, the (0,1) entry has a 1/6, the (0,2) entry has a 1/6, etc., up to the (0,6) entry. The rest of the row is all zeros. In general, the (i,j) entry of the transition matrix is the probability of moving from position "i" to position "j" on the player's next turn. When there is a chute or a ladder, things change slightly. For example, let's say our player is now on space three. From that space they have a 1/6 probability of landing on space four, a 1/6 probability of landing on space five, etc., up to space nine. But at space nine is a ladder, so rather than a 1/6 probability of landing on space nine, our player has a 1/6 probability of landing on space 29. Thus in the matrix, the (3,4) entry has a 1/6, the (3,5) entry has a 1/6, but the (3,9) entry has a zero and the (3,29) entry has a 1/6 (See Appendix 2, Pg. 21).

It should be noted here, that the transition matrix for the game of Chutes and Ladders is not a regular transition matrix, such as that described at the beginning of the paper. The reason for this is that a regular transition matrix must have all positive entries after some power n is calculated. The transition matrix for Chutes and Ladders has rows and columns of zeros that will remain no matter how many powers of the matrix are calculated.

Dr. Lund and I decided it would also be interesting to see what happens to the game lengths if a spinner labeled 1-4 is used rather than the usual 1-6 spinner. We also wanted to look at a spinner labeled 1-8 and compare this as well. I made 37x37 transition matrices for both of these possibilities as well, following the same idea as the original transition matrix (See Appendix 2, Pgs. 22 and 23).

Next, I started on the program for the smaller version with the 1-6 spinner using my Voyage

200 to program. Once I started working on the program, it was fairly easy to write. I knew that I needed some way to "move" around the board and adjust as needed for the chutes and the ladders. The Voyage 200 has a rand() function that allows the user to randomly generate numbers. I used this function to generate the numbers 1-6, simulating the 1-6 spinner. This was an essential part of the program, allowing the player to "move" between 1-6 spaces at a time. I was able to use If . . . Then loops to simulate the chutes and the ladders. At this point, the program was off and running (See Appendix 3 Pg. 36). I did some minor adjustments to write programs for the 6x6 game with both the 1-4 spinner and the 1-8 spinner (See Appendix 3 Pgs. 40 and 43).

After finishing the transition matrices and programs for the 6x6 game, I returned to the original Chutes and Ladders game. This matrix was of course going to be much larger, but the idea and process was still the same. I made a 101x101 matrix and filled in the components in the same way as the smaller matrix, again doing a transition matrix for the 1-6 spinner, 1-4 spinner, and 1-8 spinner (See Appendix 2 Pgs. 24, 28, and 32).

The programs, likewise, were very similar though slightly more complicated. I followed the same pattern I had come up with for the 6x6 version, using If . . . Then loops for the chutes and ladders. The hardest part about this was that there are 19 chutes and/or ladders in the 10x10 version as compared to only four chutes and/or ladders in the 6x6 version. The programs were much longer, but in essentially the same format and written for the 1-6 spinner, 1-4 spinner, and 1-8 spinner (See Appendix 3, Pgs. 48, 53, and 57).

Now that I had all of the programs written, I needed to run them and collect data from them. I

started with the program for the 6x6 game using a 1-4 spinner. I ran the program 500 times keeping track of the number of moves it took each time through the game. The results are in Appendix 4 on page 63. The information I really wanted to find from this data was what is the average game length. To find this I took the number of games won and multiplied it by the number of moves. I did this for each possible number of moves. I then added all of these together and divided that total by 500. For example, in the 6x6 game using a 1-4 spinner, twelve games were won in five moves, thirty-eight games were won in six moves, etc. I took twelve times five to get sixty and thirty-eight times six to get 228. I did this with all of the data and added all of the totals together. I then divided by five hundred to find a mean of 15.494, or an average game length between 15 and 16 moves.

Next I ran the program for the 6x6 game using a 1-6 spinner, again simulating the game 500 times and keeping track of the number of moves for each time played. These results are in Appendix 4 on page 64. I found the mean to be 12.056 so the average game will last about 12 moves. It made sense that the average game length for the game using a 1-6 spinner would be shorter than using the 1-4 spinner, because you have the possibility of moving further for every turn.

Finally I ran the program for the 6x6 game using the 1-8 spinner, expecting that this would have an even shorter average game length. The data collected is in Appendix 4 on page 65. The mean for this data came out to be 10.076, or an average game length of about 10 moves. Again, as expected, this was a shorter game length than the 1-6 spinner because of the possibility of moving further per spin.

Now the programs needed to be run for the 10x10 version of the game. I started again with the program simulating a 1-4 spinner, running it five hundred times and keeping track of the data (See Appendix 4, Pg. 66). I expected the mean to be higher for this game than for the 6x6 version for the obvious reason that there are 64 more spaces on the board. This was proven correct when I found the mean to be 54.152 or approximately 54 moves per game.

Next I ran the program simulating a 10x10 game using a 1-6 spinner. I ran it five hundred times, coming up with the data in Appendix 4 on page 67. I found the mean for this game to be 37.388 or about 37 moves. Again, this was lower than the mean of the 1-4 spinner due to the fact that you have a chance of moving further per spin with a 1-6 spinner than a 1-4 spinner.

Finally, I ran the program for the 10x10 game using a 1-8 spinner five hundred times. Again, I kept track of the number of moves for each game and came up with the data in Appendix 4 on page 68. Using this data, I found the mean to be 29.38 or approximately 29 moves per game. As expected, this was a shorter average game length than both the 1-4 spinner and the 1-6 spinner due to the fact that you are able to move up to eight spaces at a time rather than just four or six.

After running all of the data and finding the average game length for each of the six games, I was interested in knowing what the minimum game length was. I had a guess for all of them just from the data I had collected, but was not sure if they were completely correct, especially for the 10x10 game. Finding this information meant analyzing the transition matrices that I made earlier.

In a transition matrix, the (i,j) entry gives the probability of moving from the i-th space to the j-th space in one turn. In all of the matrices for the smaller version of the game of Chutes and Ladders, the (0,36) entry had a zero in it which makes sense, because it is impossible to get from

the "0" space to the final square in only one spin. This is the same idea as the mouse being unable to move from room one to room five in only one turn. It is impossible, so our original transition matrix has a zero in the (1,5) entry. The same held for the (0,100) entry of the larger matrices for the same reason. In order to find the minimum game length, powers of the matrices had to be calculated. This is because the same principle holds for a matrix to the second power.

In such a matrix, the (i,j) entry gives the probability of moving from i-th space to the j-th space in two turns. This idea continues for matrices to the third power, fourth power, and so on. Therefore, the minimum number of moves is found by looking at the (0,36) entry and calculating powers of the matrix. The first power that gives a non-zero entry in the (0,36) entry is going to be the minimum number of moves to win the game. Again, referring back to our mouse example, this happened after the fourth power of the transition matrix was calculated. In this matrix, we had almost a two percent chance of being in room five after four moves. Going back to our Chutes and Ladders matrices, if I calculate P^6 and find that this is the lowest power in which I have a non-zero entry in the (0,36) entry, then my minimum game length is going to be six.

After investigation of the game boards, I believe that there is no maximum game length. It is theoretically possible, though not likely, that a game could last forever. There are many loops throughout both game boards in which a player could get stuck going up a ladder and back down a chute over and over. Because there is the possibility of this happening, there is no maximum game length.

While running the programs to calculate the average game length, I kept track of the number of moves each game took. By examining this data, I was able to make a guess at each of the

minimum game lengths. For example, after running the program for the 6x6 board game using a 1-6 spinner, the lowest number of moves I came across was four. Therefore I guessed that the minimum number of moves for this particular game was four.

I used a program called Matlab to analyze the transition matrices I made and thus obtain a minimum game length. This program allowed me to input the matrices and then calculate powers of them without any difficulties. I first looked at the transition matrix for the 6x6 game with a spinner labeled 1-4. After running the program, I believed the minimum game length to be five moves. I input the 37x37 matrix and then calculated the powers of the matrix. The first power to have a non-zero entry in the (0,36) entry was the 5^{th} power proving my guess correct (See Appendix 5, Pg. 69).

Next I input the transition matrix for the 6x6 game with a 1-6 spinner. This time I thought the minimum game length was going to be four moves. After calculating powers of the matrix, the 4^{th} power was the first one in which a non-zero entry appeared in the (0,36) entry again, proving my guess correct and showing that the minimum game length was indeed four (See Appendix 5, Pg. 70).

Finally I input the transition matrix for the 6x6 game with a 1-8 spinner. My data led me to believe that the minimum game length for this game would be three moves. I used Matlab to calculate powers of the matrix and found that the first one in which a non-zero entry appeared in the (0,36) entry was the 3^{rd} power (See Appendix 5, Pg. 71). Thus the minimum game length was three and my guess was proven correct once again.

For the 10x10 version of the game, I was not as sure about my guesses. I started with the

transition matrix for the 1-4 spinner, with a guess of ten as the minimum game length. I input the matrix and calculated the powers, finding the first one with a non-zero entry in the (0,100) entry to be the 9^{th} power (See Appendix 5, Pg. 72). My guess was incorrect for this game with the minimum length being nine rather than ten.

Next I looked at the transition matrix for the 10x10 version of the game with a 1-6 spinner. My guess for the minimum length of this game was eight, but again, I was rather unsure of this. I input the transition matrix and calculated powers of the matrix, finding the 7^{th} power to be the first one in which a non-zero entry appeared in the (0,100) entry (See Appendix 5, Pg. 73). My guess was again incorrect, but off by only one number with a minimum game length of seven rather than eight.

Finally I input the transition matrix for the 10x10 version of the game with a 1-8 spinner. My guess was seven this time, but seeing as that was the minimum game length for the 10x10 version with a 1-6 spinner, I was almost 100 percent sure I was incorrect. I calculated powers of the matrix and found the 5^{th} power to be the first one in which a non-zero entry appeared in the (0,100) entry of the matrix (See Appendix 5, Pg. 74). This gives a minimum game length of five moves rather than my guess of seven moves.

The following table summarizes what I found:

Game Size	Spinner	Average Game Length	Minimum Game Length
6x6	1-4	15-16 moves	5 moves
6x6	1-6	12 moves	4 moves
6x6	1-8	10 moves	3 moves
10x10	1-4	54 moves	9 moves
10x10	1-6	37 moves	7 moves
10x10	1-8	29 moves	5 moves

At this point, I was curious as to how my data compared to other results. I knew that others have investigated the game of Chutes and Ladders, with regard to the average game length and minimum game length. I searched the Internet and found several different websites. I will summarize their results here. Dr. David Morgan found the average game length to be 35.7534 or about 36 turns (2001). This is very close to my average game length of 37 turns. Jeffery Humpherys found the average game length to be 39.2251 which is again very close to my 37 turns. He also found a minimum game length of seven moves which is exactly what I had found. Finding this information made me feel better about my data and the accuracy of it.

There are slight differences in our average game lengths, but I determined that this is most likely due to the different ways in which we found our data. Morgan simulated 10,000 games using a computer program he wrote in C source code. I only simulated 500 games and used a program I wrote for the Voyage 200. The sample size and the different code could account for the minor discrepancies between our average game lengths. Humpherys wrote a Matlab code to calculate the powers of the transition matrix up to P^{500}. He then found the probability of the game ending after one move, two moves, etc. up to 500 moves. He plotted these probabilities in a probability density function and used this information to find a minimum game length and an average game length. Again, he used a different method than Morgan or I did and this is the most likely reason for the minor discrepancies in our results.

I am a Secondary Mathematics Education Major and I am interested in finding a way in which I can incorporate this information into my classroom. My first idea was to have each of my students make their own 5x5 board (25 total spaces) and then analyze their own board by

making a transition matrix and playing the game a certain number of times. The more I thought about this idea, the more I realized there was a great potential for problems. Every single student has a different board, so there is no way to compare their data. Transition matrices may be tough for some of them to handle and they would need some help. How many times would they have to play their game in order to get an accurate average?

At this point I decided I would have them all play the same game, namely, the 6x6 board that I created and already analyzed. This way, they can compare their data not only to each other, but also to mine. I would most likely use this in a beginning statistics class to introduce the idea of mean, median, and mode. It would be a fun way for the students to start working with these ideas and the information that they yield.

The first day would be spent introducing the mean, median, and mode. This includes giving definitions and going over examples with the student. Then they would be given a worksheet to work on these concepts some more. I would also give them a brief overview of what the next few days would bring for them.

The second day I would have them actually play the game of Chutes and Ladders. Students would be paired with a partner and given a game board as well as a die and two worksheets. Each group of students would play the game 20 times, keeping track of their number of moves on a separate sheet of paper and then recording the moves of the player who won on the worksheet. After 20 games, students would find the mean, median, and mode and make guesses at a minimum and maximum game length. They would also have to prepare a short (2-3 minutes) presentation to give to the class the next day, explaining their data.

The third day would consist of the students presenting their information and then compiling everyone's data for a larger sample size. Students would come up in pairs and present their information for everyone, telling what they found for a mean, median, and mode. After everyone had presented their results, the class would have a short discussion on the differences in the data. Was it an error, or due to the fact that it's only a sample? Finally I would put up an overhead and record everyone's data sets. After we had all of that information, I would use another overheard to have the students help me find the mean, median, and mode of the larger data set as well as make another guess at the minimum and maximum game lengths. We would have another short discussion on the minimum and maximum game lengths to finish out the hour (Lesson plans, worksheets, and overheads for this three-day project can be found in Appendix 6 starting on page 75).

The lesson plans I made and the worksheets that accompany them are only meant for a three-day project. There is the possibility to take this project further. I could turn it into a week long project by having the students spend a few days on another spinner, such as a 1-4 spinner or a 1-8 spinner. Students would again be paired with a partner and play the game 20 times. They would keep track of their new data and then present this to the class as well. There could be a discussion on how the different spinner changes the average game length and minimum game lengths.

Another possibility would be to turn this into a semester long project. Pair students with a partner at the beginning of the semester. Give each pair a different spinner to investigate. One pair may use a 1-6 spinner, while another uses a 1-7 spinner, and another uses a 1-2 spinner.

Have each pair play the game 100 times while keeping track of their moves and recording the number of moves of the winner of each game. Then have the pairs of students find the average game length and make a guess at the minimum game length. Each pair of students would write a paper on their data as well as present their data to the class.

Looking at Chutes and Ladders, I found that it is not just a game that was thrown together; there is mathematics involved. There is a minimum game length and an average game length that make it such a good child's game. If the game was too long, no child would sit through the game, but with an average game length of 37 moves, the game will take 15-20 minutes and hold the child's attention. When I played this game as a child and with the kids that I babysat, I never really thought about the importance of the game having an average game length of 37. It is important, though, because it makes the game more fun for the child by holding their attention.

Another issue that I realized from this paper is that there are a number of games that could be analyzed using mathematics. Candyland and Monopoly are two that I know others have investigated, but I think that almost any game has mathematics involved and thus could be analyzed. Finding the mathematics behind a game makes the game that much more enjoyable, at least for me, because I better understand what is going on. Looking at Chutes and Ladders, I now know when I have won in the fewest possible moves and I know about how much time the game should take. Finding this information about other games is a possibility I may investigate in the future.

Reference Page

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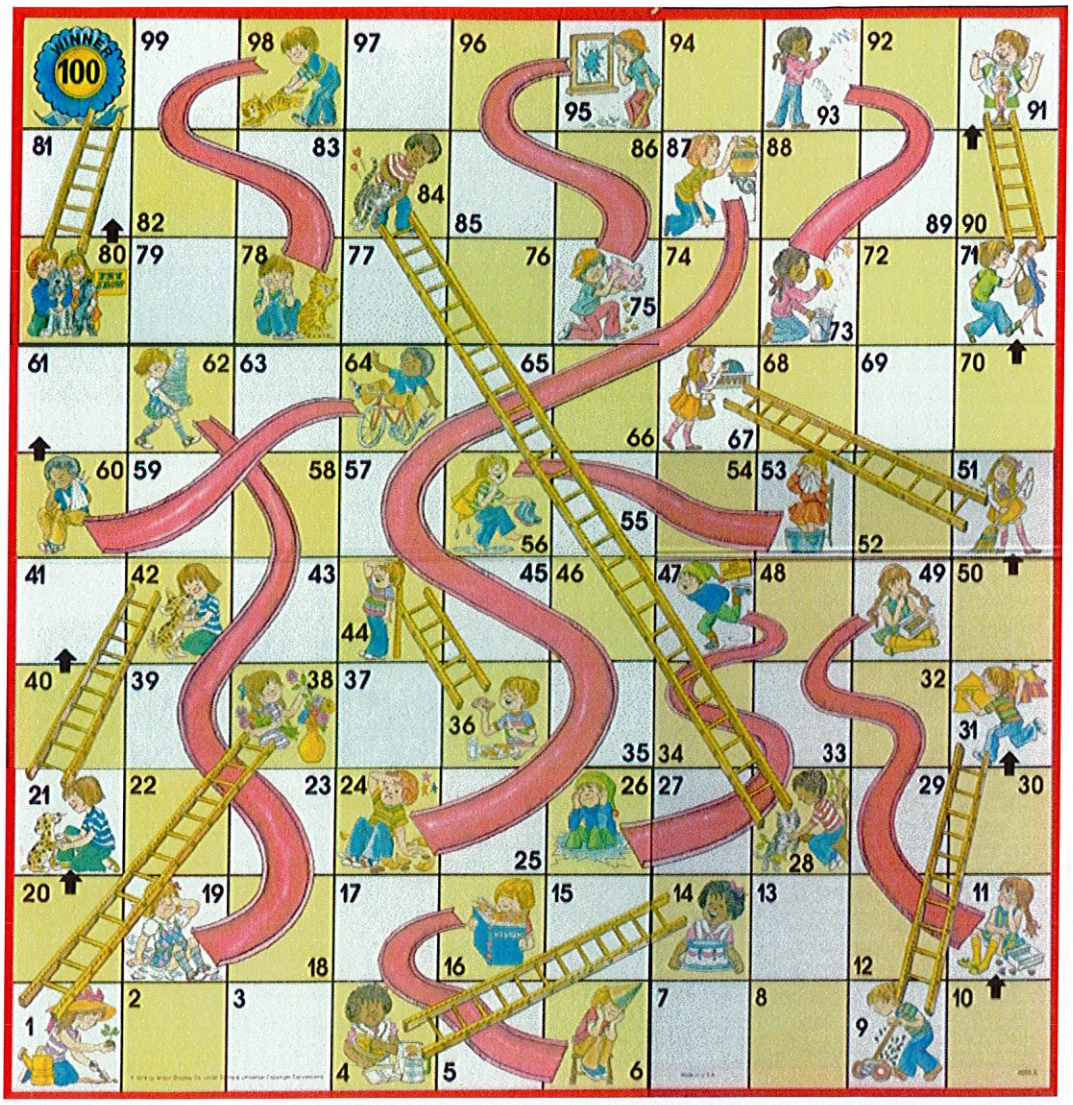
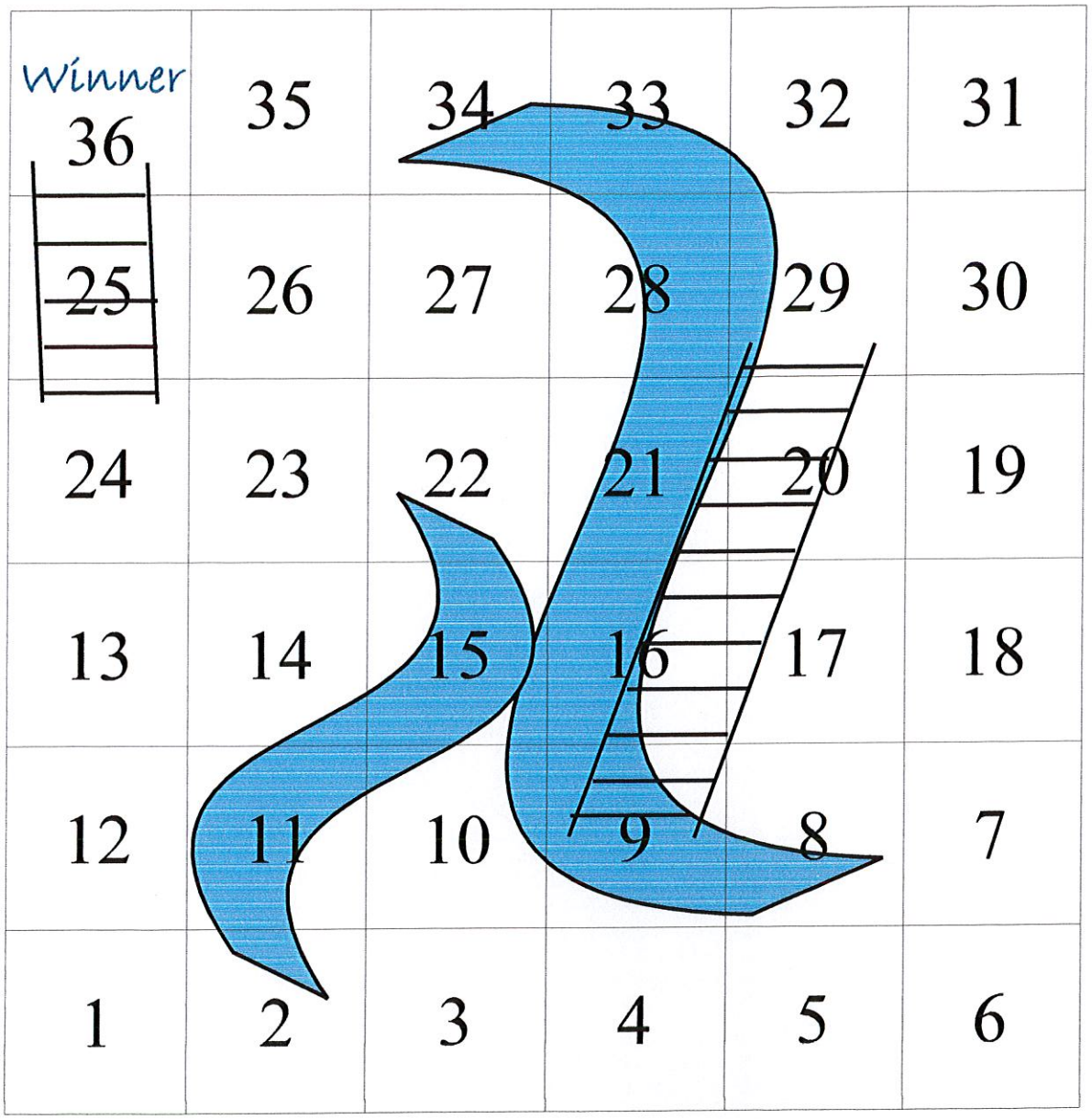
Appendix 1

Chutes and Ladders Original Game Board



Appendix 1

Smaller Version of Chutes and Ladders



Appendix 2

Transition Matrix for 6x6 Game Using 1-6 Spinner

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
4	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	1/6
19	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	1/6
20	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	1/6
21	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	1/6
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	1/6
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0
28	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0
29	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	
30	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	
31	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	
32	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2/6	1/6	0	1/6	1/6	
33	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3/6	0	1/6	1/6	
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5/6	1/6
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Appendix 2

Transition Matrix for 6x6 Game Using 1-4 Spinner

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	1/4
21	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	1/4
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	1/4
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0
30	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0
31	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0
32	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4
33	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3/4	1/4
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Appendix 2

Transition Matrix for 6x6 Game Using 1-8 Spinner

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
2	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
3	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
4	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1/8	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	1/8
17	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	0	1/8	0	0	0	0	0	0	0	0	0	0	1/8
18	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	0	1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	0	1/8
19	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	0	1/8	0	1/8	1/8	1/8	0	0	0	0	0	0	0	0	1/8
20	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	1/8
21	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	1/8
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	1/8
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0
26	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0
27	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	0
28	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	1/8
29	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	1/8	1/8
30	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2/8	1/8	1/8	1/8	0	1/8	1/8
31	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3/8	1/8	1/8	0	1/8	1/8
32	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4/8	1/8	0	1/8	1/8
33	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5/8	0	1/8	1/8
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7/8	1/8
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Appendix 2

Transition Matrix for 10x10 Game Using 1-6 Spinner

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
0	0	0	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	1/6	1/6	1/6	1/6	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1/6	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	1/6	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	1/6	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	1/6	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	1/6	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	1/6	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	1/6	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	1/6	0	0	1/6	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	1/6	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	0
47	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2

51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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Appendix 2

50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1/6	2/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1/6	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1/6	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1/6	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1/6	2/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2/6	1/6	0	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/6	0	0	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	0	1/6	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	0	1/6	1/6	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2/6	1/6	0	1/6	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3/6	0	1/6	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5/6	1/6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Appendix 2

Transition Matrix for 10x10 Game Using 1-4 Spinner

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
0	0	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	1/4	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	1/4	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	0	0	0	0	0	1/4	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	1/4	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2

51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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83	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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Appendix 2

50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2

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0	0	0	1/4	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	1/4	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/4	0	0	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	1/4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	1/4	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	1/4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/4	0	1/4	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3/4	1/4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Appendix 2

Transition Matrix for 10x10 Game Using 1-8 Spinner

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1/8	0	1/8	1/8	1/8	1/8	0	1/8	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	1/8	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1/8	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1/8	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1/8	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	1/8	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	1/8	0	0	0	0	0	0	1/8	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0
15	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	1/8	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	1/8	1/8	0	0	0	0	0	1/8	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	0	1/8	1/8	1/8	0	0	0	0	1/8	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	0	1/8	1/8	1/8	1/8	1/8	0	0	1/8	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	1/8	1/8	1/8	1/8	1/8	1/8	0	1/8	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	1/8	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	1/8	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	1/8	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	1/8	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	1/8	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2

51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
94	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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97	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2

50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/8	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/8	0	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/8	0	1/8	1/8	1/8	1/8	0	0	0	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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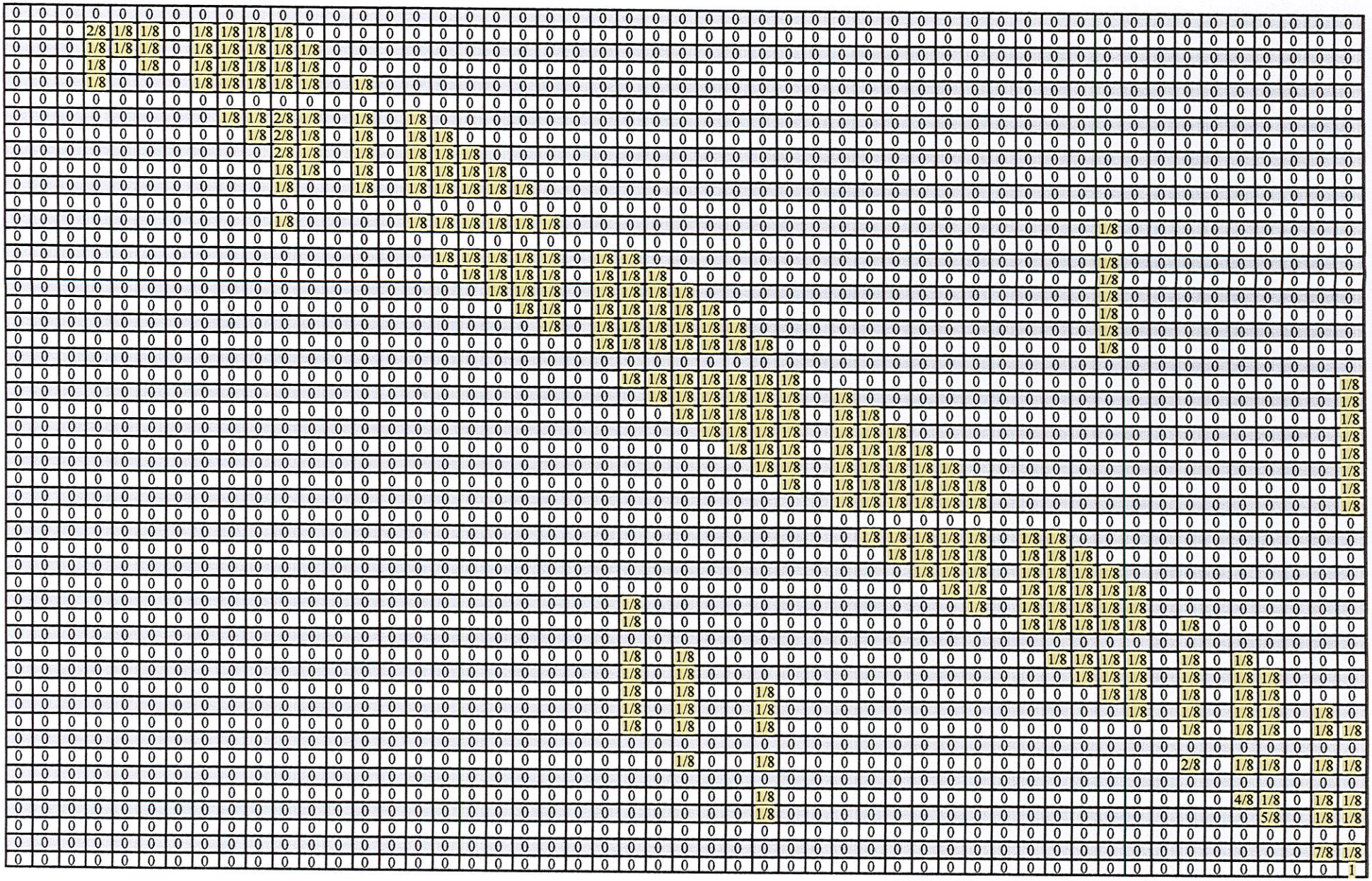
0	0	1/8	2/8	1/8	1/8	0	1/8	1/8	0	0	0	0	0	0	0	0	1/8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2



Appendix 3

Program for 6x6 Game Using 1-6 Spinner

```
chutes66()
Prgm

Lbl begin

ClrIO

Disp "        A program to model"
Disp "        a 36 space game of"
Disp "        Chutes and Ladders"
Disp "       using a 1-6 spinner."

Pause

DelVar move, k, count, movelist

0->count
0->k
{0}->movelist

Lbl top

While k<31

  rand(6)->move
  k+move->k

  If k=9 Then
    29->k
  EndIf

  If k=22 Then
    2->k
  EndIf

  If k=24 Then
    36->k
  EndIf

  If k=34 Then
```

Appendix 3

```
      8->k
    EndIf

    count+1->count
    k->movelist[count]

  EndWhile

  While k=31

    rand(6)->move

    If move<6 Then
      k+move->k
      count+1->count
      k->movelist[count]

      If k=34 Then
        8->k
        k->movelist[count]
        Goto top
      EndIf

    EndIf

EndWhile

While k=32

  rand(6)->move

  If move<5 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf
```

Appendix 3

```
  EndIf

EndWhile

While k=33

  rand(6)->move

  If move<4 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile

While k=35

  rand(6)->move

  If move<2 Then
    k+move->k
    count+1->count
    k->movelist[count]
EndIf

EndWhile

ClrIO

Disp "Congratulations!"
Disp "You won in"
Disp count
Disp "moves!"
```

Appendix 3

```
Pause movelist

ClrIO

Disp "Do you wish to run the program again?"
Disp "1 for yes, 2 for no"

Prompt reply

If reply=1 Then
  Goto begin
Else
  Disp "You are finished!"
EndIf

End Prgm
```

Appendix 3

Program for 6x6 Game Using 1-4 Spinner

```
chutes64()
Prgm

Lbl begin

ClrIO

Disp "        A program to model"
Disp "        a 36 space game of"
Disp "        Chutes and Ladders"
Disp "       using a 1-4 spinner."

Pause

DelVar move, k, count, movelist

0->count
0->k
{0}->movelist

Lbl top

While k<33

  rand(4)->move
  k+move->k

  If k=9 Then
    29->k
  EndIf

  If k=22 Then
    2->k
  EndIf

  If k=24 Then
    36->k
  EndIf

  If k=34 Then
```

Appendix 3

```
    8->k
  EndIf

  count+1->count
  k->movelist[count]

EndWhile

While k=33

  rand(4)->move

  If move<4 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile

While k=35

  rand(4)->move

  If move<2 Then
    k+move->k
    count+1->count
    k->movelist[count]
EndIf

EndWhile

ClrIO

Disp "Congratulations!"
```

Appendix 3

```
Disp "You won in"
Disp count
Disp "moves!"

Pause movelist

ClrIO

Disp "Do you wish to run the program again?"
Disp "1 for yes, 2 for no"

Prompt reply

If reply=1 Then
  Goto begin
Else
  Disp "You are finished!"
EndIf

End Prgm
```

Appendix 3

Program for 6x6 Game Using 1-8 Spinner

```
chutes68()
Prgm

Lbl begin

ClrIO

Disp "        A program to model"
Disp "        a 36 space game of"
Disp "        Chutes and Ladders"
Disp "       using a 1-8 spinner."

Pause

DelVar move, k, count, movelist

0->count
0->k
{0}->movelist

Lbl top

While k<29

  rand(8)->move
  k+move->k

  If k=9 Then
    29->k
  EndIf

  If k=22 Then
    2->k
  EndIf

  If k=24 Then
    36->k
  EndIf

  If k=34 Then
```

Appendix 3

```
    8->k
  EndIf

  count+1->count
  k->movelist[count]

EndWhile

While k=29

  rand(8)->move

  If move<8 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile

While k=30

  rand(8)->move

  If move<7 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf
```

Appendix 3

```
  EndIf

EndWhile

While k=31

  rand(8)->move

  If move<6 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile

While k=32

  rand(8)->move

  If move<5 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile
```

Appendix 3

```
While k=33

  rand(8)->move

  If move<4 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=34 Then
      8->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile

While k=35

  rand(8)->move

  If move<2 Then
    k+move->k
    count+1->count
    k->movelist[count]
EndIf

EndWhile

ClrIO

Disp "Congratulations!"
Disp "You won in"
Disp count
Disp "moves!"

Pause movelist

ClrIO
```

Appendix 3

```
Disp "Do you wish to run the program again?"
Disp "1 for yes, 2 for no"

Prompt reply

If reply=1 Then
  Goto begin
Else
  Disp "You are finished!"
EndIf

End Prgm
```

Appendix 3

Program for 10x10 Game Using 1-6 Spinner

```
chute106()
Prgm

Lbl begin

ClrIO

Disp "        A program to model"
Disp "        a 100 space game of"
Disp "         Chutes and Ladders"
Disp "         using a 1-6 spinner."

Pause

DelVar move, k, count, movelist

0->count
0->k
{0}->movelist

Lbl top

While k<95

  rand(6)->move
  k+move->k

  If k=1 Then
    38->k
  EndIf

  If k=4 Then
    14->k
  EndIf

  If k=9 Then
    31->k
  EndIf

  If k=16 Then
```

Appendix 3

```
  6->k
EndIf

If k=21 Then
  42->k
EndIf

If k=28 Then
  84->k
EndIf

If k=36 Then
  44->k
EndIf

If k=48 Then
  26->k
EndIf

If k=49 Then
  11->k
EndIf

If k=51 Then
  67->k
EndIf

If k=56 Then
  53->k
EndIf

If k=62 Then
  19->k
EndIf

If k=64 Then
  60->k
EndIf

If k=71 Then
  91->k
EndIf
```

Appendix 3

```
  If k=80 Then
    100->k
  EndIf

  If k=87 Then
    24->k
  EndIf

  If k=93 Then
    73->k
  EndIf

  If k=95 Then
    75->k
  EndIf

  If k=98 Then
    78->k
  EndIf

  count+1->count
  k->movelist[count]

EndWhile

While k=96

  rand(6)->move

  If move<5 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=98 Then
      78->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf
```

Appendix 3

```
EndWhile

While k=97

  rand(6)->move

  If move<4 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=98 Then
      78->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf

EndWhile

While k=99

  rand(6)->move

  If move<2 Then
    k+move->k
    count+1->count
    k->movelist[count]
  EndIf

EndWhile

ClrIO

Disp "Congratulations!"
Disp "You won in"
Disp count
Disp "moves!"

Pause movelist
```

Appendix 3

```
ClrIO

Disp "Do you wish to run the program again?"
Disp "1 for yes, 2 for no"

Prompt reply

If reply=1 Then
  Goto begin
Else
  Disp "You are finished!"
EndIf

EndPrgm
```

Appendix 3

Program for 10x10 Game Using 1-4 Spinner

```
chute104()
Prgm

Lbl begin

ClrIO

Disp "        A program to model"
Disp "        a 100 space game of"
Disp "         Chutes and Ladders"
Disp "         using a 1-4 spinner."

Pause

DelVar move, k, count, movelist

0->count
0->k
{0}->movelist

Lbl top

While k<97

  rand(4)->move
  k+move->k

  If k=1 Then
    38->k
  EndIf

  If k=4 Then
    14->k
  EndIf

  If k=9 Then
    31->k
  EndIf

  If k=16 Then
```

Appendix 3

```
  6->k
EndIf

If k=21 Then
  42->k
EndIf

If k=28 Then
  84->k
EndIf

If k=36 Then
  44->k
EndIf

If k=48 Then
  26->k
EndIf

If k=49 Then
  11->k
EndIf

If k=51 Then
  67->k
EndIf

If k=56 Then
  53->k
EndIf

If k=62 Then
  19->k
EndIf

If k=64 Then
  60->k
EndIf

If k=71 Then
  91->k
EndIf
```

Appendix 3

```
  If k=80 Then
    100->k
  EndIf

  If k=87 Then
    24->k
  EndIf

  If k=93 Then
    73->k
  EndIf

  If k=95 Then
    75->k
  EndIf

  If k=98 Then
    78->k
  EndIf

  count+1->count
  k->movelist[count]

EndWhile

While k=97

  rand(4)->move

  If move<4 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=98 Then
      78->k
      k->movelist[count]
      Goto top
    EndIf

  EndIf
```

Appendix 3

```
EndWhile

While k=99

  rand(4)->move

  If move<2 Then
    k+move->k
    count+1->count
    k->movelist[count]
  EndIf

EndWhile

ClrIO

Disp "Congratulations!"
Disp "You won in"
Disp count
Disp "moves!"

Pause movelist

ClrIO

Disp "Do you wish to run the program again?"
Disp "1 for yes, 2 for no"

Prompt reply

If reply=1 Then
  Goto begin
Else
  Disp "You are finished!"
EndIf

EndPrgm
```

Appendix 3

Program for 10x10 Game Using 1-8 Spinner

```
chute108()
Prgm

Lbl begin

ClrIO

Disp "        A program to model"
Disp "        a 100 space game of"
Disp "         Chutes and Ladders"
Disp "         using a 1-8 spinner."

Pause

DelVar move, k, count, movelist

0->count
0->k
{0}->movelist

Lbl top

While k<93

 rand(8)->move
 k+move->k

 If k=1 Then
   38->k
 EndIf

 If k=4 Then
   14->k
 EndIf

 If k=9 Then
   31->k
 EndIf

 If k=16 Then
```

Appendix 3

```
  6->k
EndIf

If k=21 Then
  42->k
EndIf

If k=28 Then
  84->k
EndIf

If k=36 Then
  44->k
EndIf

If k=48 Then
  26->k
EndIf

If k=49 Then
  11->k
EndIf

If k=51 Then
  67->k
EndIf

If k=56 Then
  53->k
EndIf

If k=62 Then
  19->k
EndIf

If k=64 Then
  60->k
EndIf

If k=71 Then
  91->k
EndIf
```

Appendix 3

```
    If k=80 Then
      100->k
    EndIf

    If k=87 Then
      24->k
    EndIf

    If k=93 Then
      73->k
    EndIf

    If k=95 Then
      75->k
    EndIf

    If k=98 Then
      78->k
    EndIf

    count+1->count
    k->movelist[count]

EndWhile

While k=94

  rand(8)->move

  If move<7 Then
    k+move->k
    count+1->count
    k->movelist[count]

    If k=95 Then
      75->k
      k->movelist[count]
      Goto top
    EndIf

    If k=98 Then
      78->k
```

Appendix 3

```
        k->movelist[count]
        Goto top
      EndIf

    EndIf

  EndWhile

  While k=96

    rand(8)->move

    If move<5 Then
      k+move->k
      count+1->count
      k->movelist[count]

      If k=98 Then
        78->k
        k->movelist[count]
        Goto top
      EndIf

    EndIf

  EndWhile

  While k=97

    rand(8)->move

    If move<4 Then
      k+move->k
      count+1->count
      k->movelist[count]

      If k=98 Then
        78->k
        k->movelist[count]
        Goto top
      EndIf
```

Appendix 3

```
  EndIf

EndWhile

While k=99

  rand(8)->move

  If move<2 Then
    k+move->k
    count+1->count
    k->movelist[count]
  EndIf

EndWhile

ClrIO

Disp "Congratulations!"
Disp "You won in"
Disp count
Disp "moves!"

Pause movelist

ClrIO

Disp "Do you wish to run the program again?"
Disp "1 for yes, 2 for no"

Prompt reply

If reply=1 Then
  Goto begin
Else
  Disp "You are finished!"
EndIf

EndPrgm
```

Appendix 4

Explanation of Bar Graphs

The following Bar Graphs show the data collected from running the different programs. The x-axis gives the possible number of moves it took to win a game and the y-axis gives the number of games won in x amount of moves.

The mean was found by taking the number of games won in x amount of moves and multiplying that by the number of moves. For example, let us say that six games were won in seven moves. We would take six multiplied by seven to get 42. I continued this process for all of the possible number of moves and then added all of these numbers together. I took this large number and divided it by 500 (the number of times the game was "played") to find my average number of moves or the mean.

To find the median, I looked at the number of moves that were used to win a game. I had to first order these from least to greatest and then find the middle two numbers and divide by two. Ordering these consisted of looking at how many games were won by each number of moves. For example, again, if six games were won in seven moves, I would have seven sixes in my list. Overall, there were 500 numbers in this list (because 500 games were "played") ranging from the least number of moves it took to win a game to the most number of moves it took to win a game.

To find the mode, I found the number of moves it took to win a game that had actually won the most games. For example, if I had won 60 games in ten moves and this was the most number of games won, then ten would be my mode.

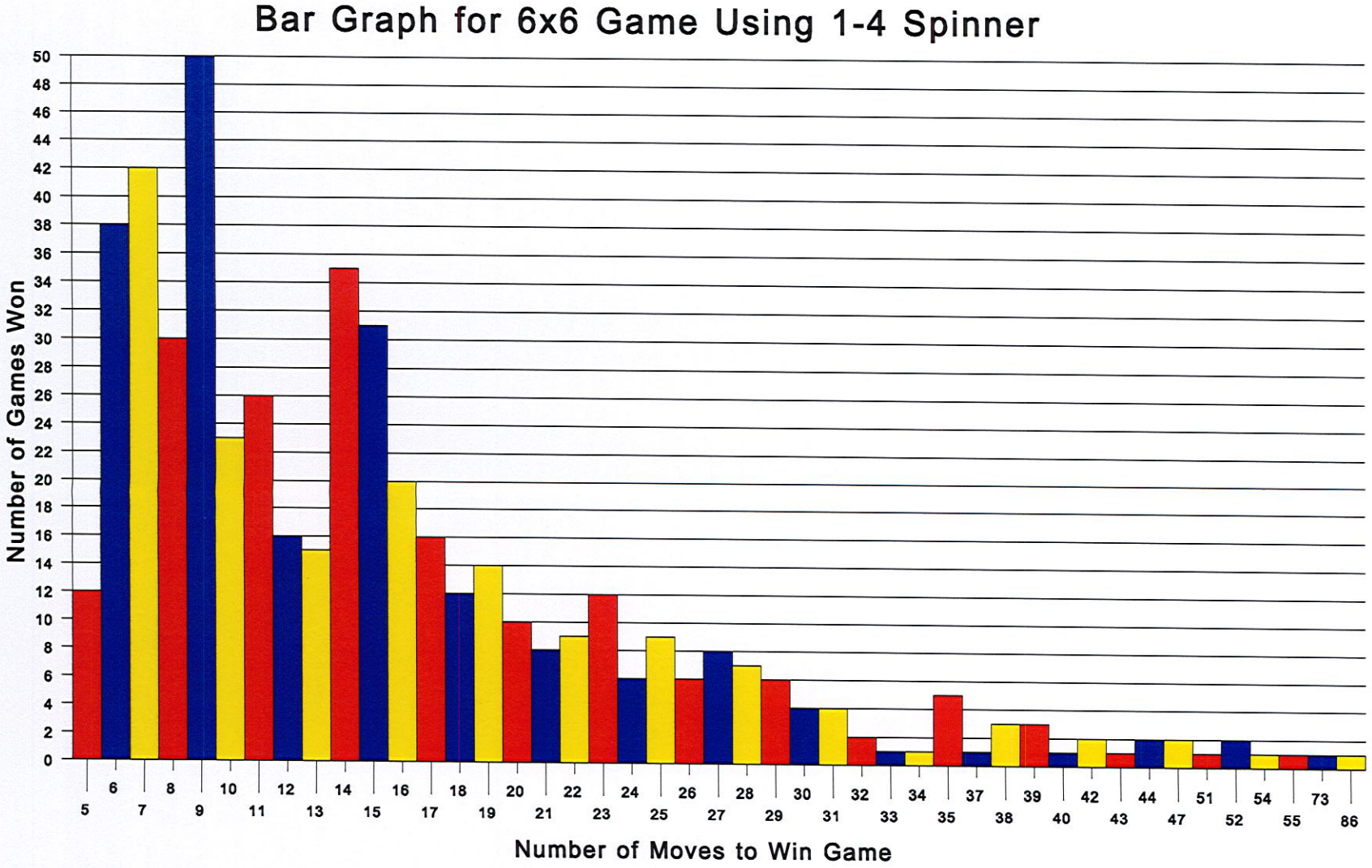
Appendix 4

This is the bar graph for the 6x6 game with a 1-4 spinner after a simulation of 500 games. The x-axis gives the number of moves it took to win a game and the y-axis is the number of games that were won. For example, 12 games were won in 5 moves.

Mean: 15.494

Median: 13

Mode: 9



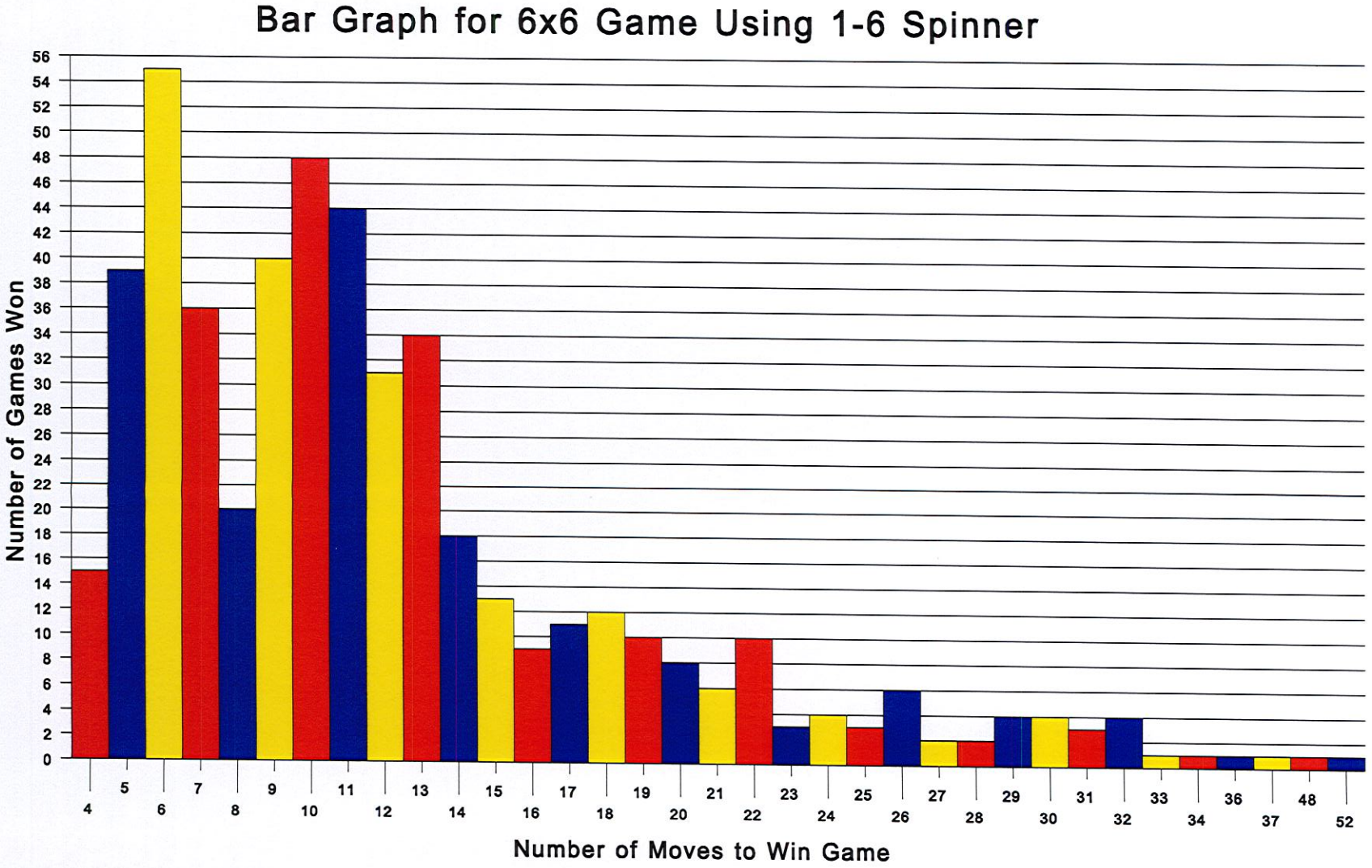
Appendix 4

This is the bar graph for the 6x6 game with a 1-6 spinner after a simulation of 500 games. The x-axis gives the number of moves it took to win a game and the y-axis is the number of games that were won. For example, 15 games were won in 4 moves.

Mean: 12.056

Median: 10

Mode: 6



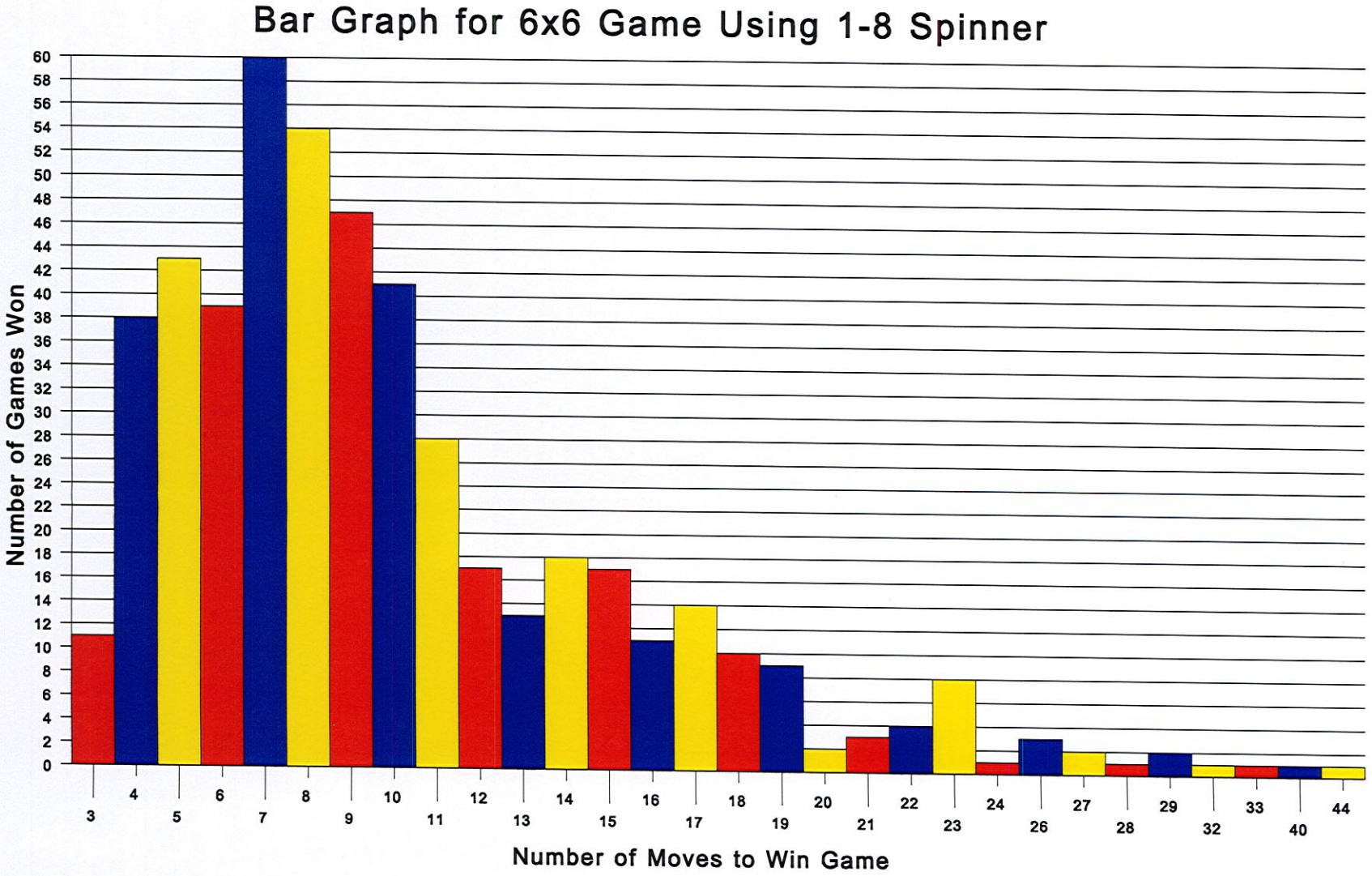
Appendix 4

This is the bar graph for the 6x6 game with a 1-8 spinner after a simulation of 500 games. The x-axis gives the number of moves it took to win a game and the y-axis is the number of games that were won. For example, 11 games were won in 3 moves.

Mean: 10.076

Median: 9

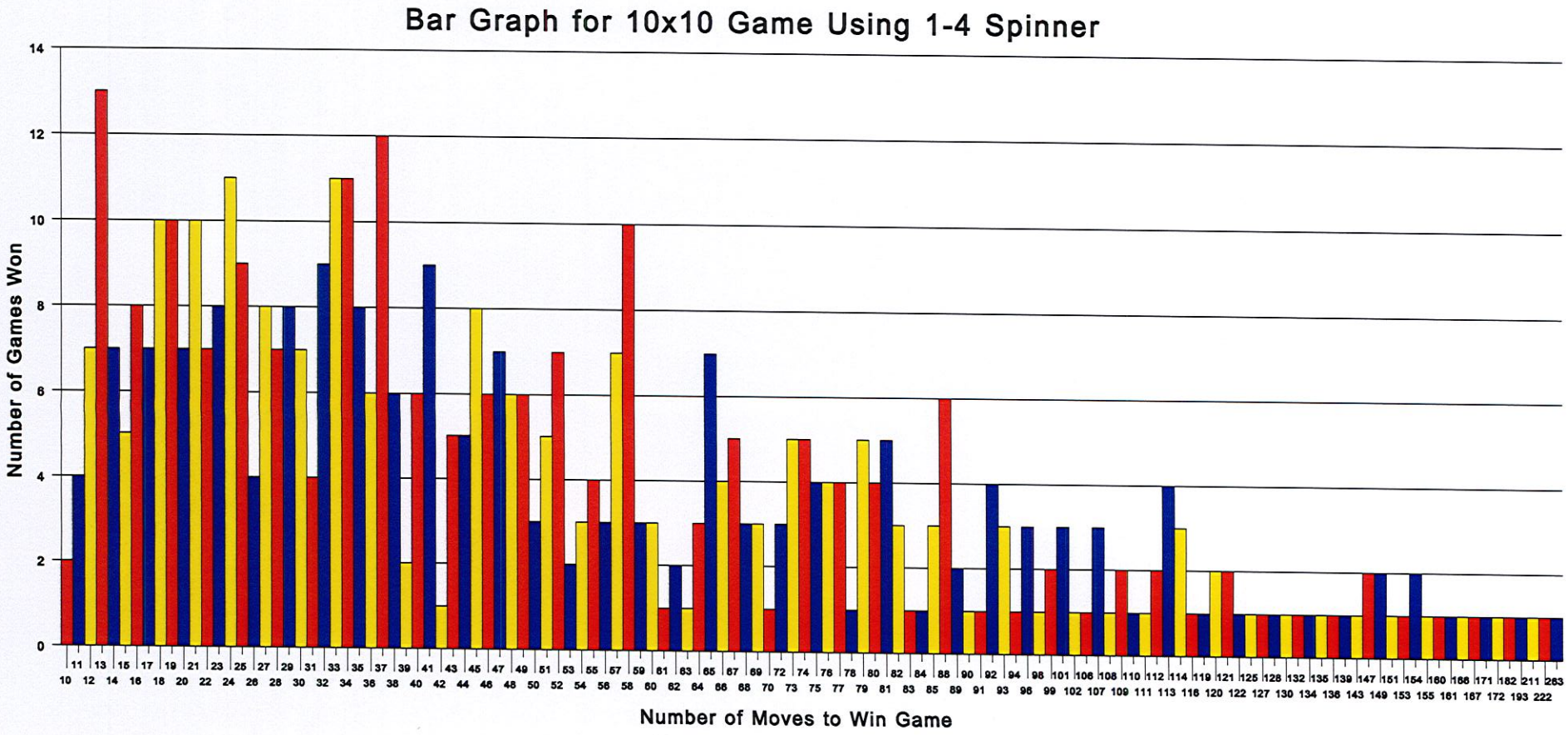
Mode: 7



Appendix 4

This is the bar graph for the 10x10 game with a 1-4 spinner after a simulation of 500 games. The x-axis gives the number of moves it took to win a game and the y-axis is the number of games that were won. For example, 2 games were won in 10 moves.

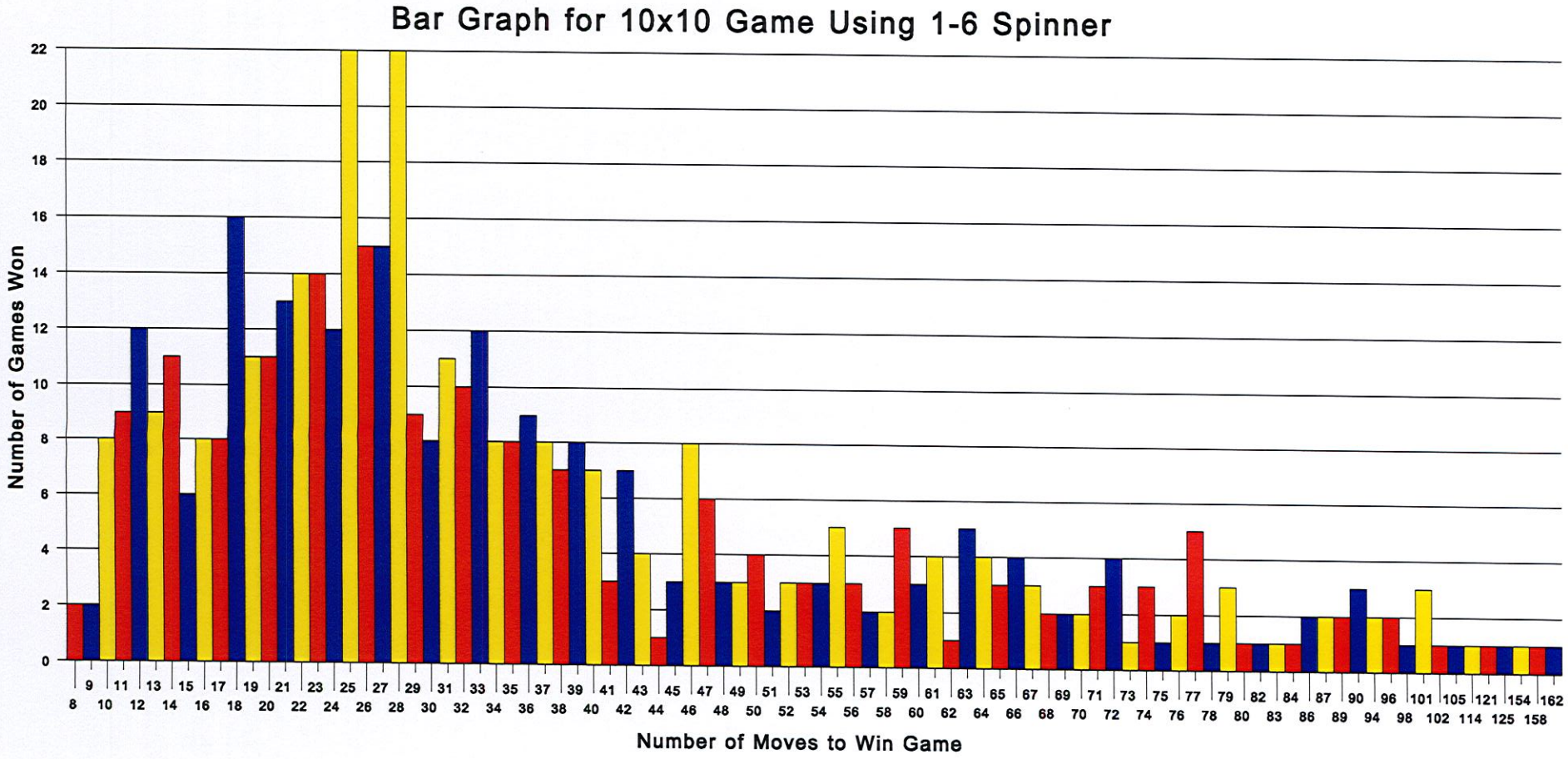
Mean: 54.152
Median: 44
Mode: 13



Appendix 4

This is the bar graph for the 10x10 game with a 1-6 spinner after a simulation of 500 games. The x-axis gives the number of moves it took to win a game and the y-axis is the number of games that were won. For example, 2 games were won in 8 moves.

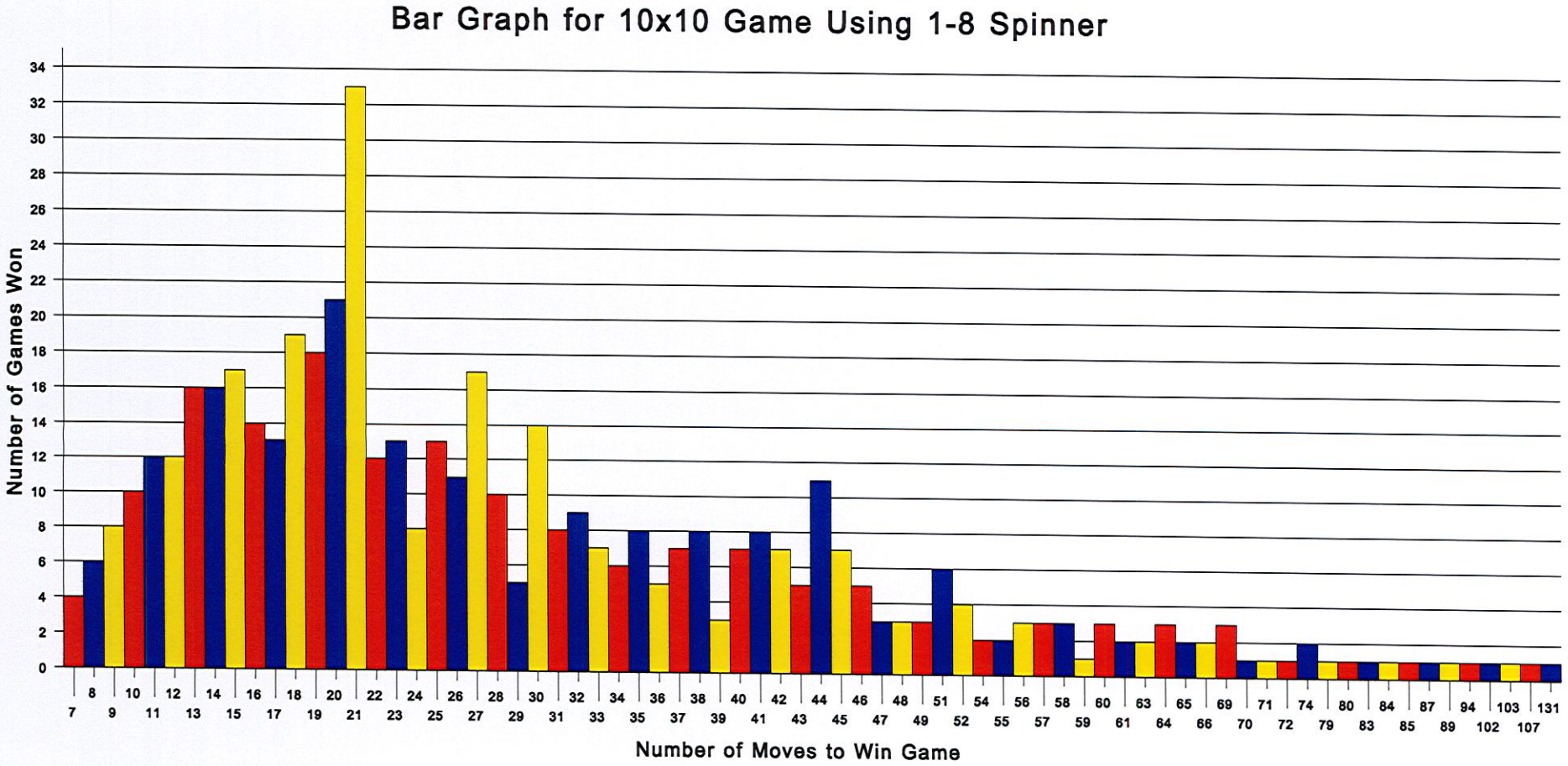
Mean: 37.388
Median: 30
Mode: 28



Appendix 4

This is the bar graph for the 10x10 game with a 1-8 spinner after a simulation of 500 games. The x-axis gives the number of moves it took to win a game and the y-axis is the number of games that were won. For example, 4 games were won in 7 moves.

Mean: 29.38
Median: 24
Mode: 21



Appendix 5

This is the last column of the matrix for the 6x6 game using a spinner labeled 1-4. This is to the 5^{th} power and it's the first one in which there is a number in the (0,36) entry. Thus the minimum number of moves for this game is 5.

Column 37

0.01953125000000
0.04003906250000
0.05664062500000
0.06933593750000
0.07910156250000
0.14648437500000
0.13476562500000
0.13281250000000
0.14355468750000
0
0.13574218750000
0.17578125000000
0.21582031250000
0.24707031250000
0.25585937500000
0.25878906250000
0.28808593750000
0.29687500000000
0.24218750000000
0.26074218750000
0.41308593750000
0.37402343750000
0
0.40625000000000
0
0.28222656250000
0.30761718750000
0.33984375000000
0.39062500000000
0.42968750000000
0.37011718750000
0.44628906250000
0.56738281250000
0.56738281250000
0
0.76269531250000
1.00000000000000

Appendix 5

This is the last column of the matrix for the 6x6 game using a spinner labeled 1-6. This is to the 4^{th} power and it's the first one in which there is a number in the (0,36) entry. Thus the minimum number of moves for this game is 4.

Column 37

0.01620370370370
0.02237654320988
0.03086419753086
0.06635802469136
0.07330246913580
0.08410493827160
0.10108024691358
0.12191358024691
0.14429012345679
0
0.15740740740741
0.17361111111111
0.20293209876543
0.22145061728395
0.23225308641975
0.23842592592593
0.20833333333333
0.21913580246914
0.33487654320988
0.31712962962963
0.30864197530864
0.30709876543210
0
0.33950617283951
0
0.26388888888889
0.29012345679012
0.31172839506173
0.28549382716049
0.31635802469136
0.42361111111111
0.42361111111111
0.42361111111111
0.42361111111111
0
0.51774691358025
1.00000000000000

Appendix 5

This is the last column of the matrix for the 6x6 game using a spinner labeled 1-8. This is to the 3rd power and it's the first one in which there is a number in the (0,37) component. Thus the minimum number of moves for this game is 3.

Column 37

0.01757812500000
0.04296875000000
0.04687500000000
0.05273437500000
0.06054687500000
0.07031250000000
0.08007812500000
0.09179687500000
0.11523437500000
0
0.13085937500000
0.14453125000000
0.15625000000000
0.16601562500000
0.15625000000000
0.16601562500000
0.26367187500000
0.25000000000000
0.23828125000000
0.23046875000000
0.23632812500000
0.24218750000000
0
0.27148437500000
0
0.18945312500000
0.18164062500000
0.19726562500000
0.29492187500000
0.29492187500000
0.29492187500000
0.29492187500000
0.29492187500000
0.29492187500000
0
0.33007812500000
1.00000000000000

Appendix 5

This is the last column of the matrix for the 10x10 game using a spinner labeled 1-4. This is to the 9th power and it's the first one in which there is a number in the (0,100) entry. Thus the minimum number of moves for this game is 9.

Column 101

0.00088119506836	0.01502227783203	0.41989517211914
0	0.01562118530273	0.41940307617188
0.00001907348633	0	0.41719436645508
0.00001907348633	0.01070404052734	0
0	0.01594543457031	0.42772293090820
0.00032043457031	0.02516937255859	0.43486785888672
0.00032043457031	0.03404235839844	0.43361663818359
0.00032043457031	0.04049682617188	0.42718887329102
0.00032043457031	0.04861068725586	0.54918670654297
0	0.06983184814453	0.48875808715820
0.00002288818359	0.07315063476563	0.44860076904297
0.00007247924805	0.06227111816406	0.41100692749023
0.00007247924805	0.07291030883789	0
0.00073242187500	0.14382934570313	0.25331115722656
0.00196075439453	0	0.27925109863281
0.00389862060547	0	0.24735641479492
0	0.10672378540039	0.29399490356445
0.01379394531250	0	0.32866668701172
0.02081680297852	0.00864028930664	0.35857009887695
0.02845001220703	0.02004241943359	0
0.04432678222656	0.02977371215820	0.46030426025391
0	0.04854202270508	0.45251083374023
0.06331253051758	0	0.47019195556641
0.06709671020508	0.09747314453125	0.45600128173828
0.11266326904297	0.08779144287109	0.51411819458008
0.09498977661133	0.13787460327148	0
0.07970428466797	0.13787460327148	0.55989837646484
0.06661987304688	0.19798278808594	0
0	0	0.75634002685547
0.00377655029297	0.32726669311523	0.75634002685547
0.00502395629883	0	0
0.00618743896484	0.41201019287109	0.92491531372070
0.01674652099609	0.41338348388672	1.00000000000000
0.01564788818359	0.42037582397461	

Appendix 5

This is the last column of the matrix for the 10x10 game using a spinner labeled 1-6. This is to the 7th power and it's the first one in which there is a number in the (0,100) entry. Thus the minimum number of moves for this game is 7.

Column 101

0.00156464334705	0.01369241540924	0.33283321902149
0	0.01715392089620	0.33896676383173
0.00030006858711	0	0.34124585619570
0.00062157064472	0.02007958962048	0
0	0.02373042409694	0.32668895747599
0.00036436899863	0.03257887517147	0.32965034865112
0.00047153635117	0.03931970164609	0.43153792295382
0.00066443758573	0.04422439414723	0.40024862825789
0.00096450617284	0.04561756973022	0.37870799039781
0	0.04139160379515	0.36450474394147
0.00118955761317	0.04469593049840	0.35611711248285
0.00285779606767	0.08327974965706	0.35223408207590
0.00496899291267	0.07537437128487	0
0.00745884773663	0.06991955304070	0.21452760631001
0.01026306012803	0	0.24234467878372
0.01423182441701	0	0.26512845793324
0	0.07375971650663	0.28797296524920
0.02823145290352	0	0.31049597050754
0.03300040009145	0.04877543438500	0.32670324645633
0.03675482967535	0.06483267604024	0
0.03955904206676	0.08384773662551	0.39521533493370
0	0.10460605281207	0.39355781321445
0.07423482510288	0	0.42336105395519
0.06667595450389	0.13135502400549	0.44921696387746
0.06001014517604	0.14190386374028	0.43410636716964
0.05389446159122	0.17225365797897	0
0.04830747027892	0.19766303726566	0.57557441700960
0.04330275491541	0.21821059099223	0
0	0	0.61161479766804
0.00267918381344	0.27925668724280	0.61161479766804
0.00727309099223	0	0
0.00789466163695	0.32508144718793	0.72091835276635
0.00883773433928	0.32263803155007	1.00000000000000
0.01084533607682	0.31993384202103	

Appendix 5

This is the last column of the matrix for the 10x10 game using a spinner labeled 1-8. This is to the 5th power and it's the first one in which there is a number in the (0,100) entry. Thus the minimum number of moves for this game is 5.

Column 101

0.00054931640625	0.00671386718750	0.23895263671875
0	0.00955200195313	0.23818969726563
0.00018310546875	0	0.23760986328125
0.00018310546875	0.01202392578125	0
0	0.01425170898438	0.31561279296875
0.00018310546875	0.01623535156250	0.29592895507813
0.00024414062500	0.01702880859375	0.28140258789063
0.00033569335938	0.01638793945313	0.27139282226563
0.00033569335938	0.01828002929688	0.26623535156250
0	0.04150390625000	0.26397705078125
0.00054931640625	0.03945922851563	0.26470947265625
0.00076293945313	0.03814697265625	0.24301147460938
0.00231933593750	0.03695678710938	0
0.00305175781250	0.03695678710938	0.16952514648438
0.00454711914063	0	0.18658447265625
0.00598144531250	0	0.20306396484375
0	0.04614257812500	0.22686767578125
0.00878906250000	0	0.23620605468750
0.01004028320313	0.03924560546875	0.25616455078125
0.01123046875000	0.05111694335938	0
0.03082275390625	0.04980468750000	0.29916381835938
0	0.06570434570313	0.31402587890625
0.02874755859375	0	0.30691528320313
0.02734375000000	0.09494018554688	0.32443237304688
0.02597045898438	0.11367797851563	0.40444946289063
0.02462768554688	0.13092041015625	0
0.02331542968750	0.14614868164063	0.41781616210938
0.02215576171875	0.15905761718750	0
0	0	0.44219970703125
0.00418090820313	0.20043945312500	0.44219970703125
0.00454711914063	0	0
0.00503540039063	0.23342895507813	0.48709106445313
0.00555419921875	0.23767089843750	1.00000000000000
0.00607299804688	0.23910522460938	

Appendix 6

Lesson Plan

Date: Day 1

Class: Statistics

Hour:

Learner Objectives:

Students will learn about mean, median, and mode. They will learn the definition as well as what each tells them about their data.

Method:

Lecture
Examples
Worksheet

Content:

Start with this on the board: Lisa scores 7, 10, 15, 2, 15, and 6 points in her basketball games.
Put this into a list {7, 10, 15, 2, 15, 6}
What is her average score?

Have students work at desks...explain that they just found the mean.

Mean = average
To find...add all the numbers in the list together and divide by the total number of entries in the list

Median = "number in the middle"
To find...order numbers in list from smallest to largest and choose the middle one. If even number of entries, take the two middle numbers and divide by two.

Mode = entry occurring most often in the list

Do some more examples (out of book or made up)

Hand out assignment and give preview of project for the next day.

Appendix 6

Name _____

Mean, Median, and Mode

Remember:

Mean = average; add numbers in list together and divide by total number of entries in the list

Median = "number in the middle"; if even number of entries, take the two in the middle, add them, and divide by two

Mode = number that occurs most often in the list

Find the **mean, median,** and **mode** of the following sets of data.

1) 135, 120, 116, 119, 121, 125, 135, 131, 123

2) 89, 78, 91, 82, 75, 89, 84, 95, 89, 93

3) 3.15, 3.62, 2.54, 2.81, 3.97, 1.85, 1.93, 2.63, 2.50, 2.80

4) 100, 78, 93, 84, 91, 100, 82, 79

Appendix 6

5) 80, 90, 90, 100, 85, 90

6) 140, 220, 180, 90, 140, 200

7) 93, 84, 97, 98, 100, 78, 86, 100, 85, 92, 72, 55, 91, 90, 75, 94, 83, 60, 81, 95

8) 6.98, 55.32, 75.09, 155.67, 122.28

9) 13, 9, 32, 97, 33, 82, 27, 50, 45, 61, 3, 61, 98, 41, 20

10) 77, 89, 68, 100, 89, 97, 76, 92, 84

Appendix 6

11) Some teenagers are comparing the number of times they have been to the movies in the past year. The following table illustrates how many times each person went to the movie theater in each month.

	Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
John	1	3	2	5	2	3	1	4	2	3	2	1
Mary	1	2	1	1	1	3	3	2	2	4	1	2
Brian	1	3	2	2	1	4	5	3	2	2	1	3
Kelly	2	2	1	1	3	2	4	1	3	2	3	2

1. Put each teenagers movies per month into a list. Then find the mean, median and mode of each list. **For example**:

John {1, 3, 2, 5, 2, 3, 1, 4, 2, 3, 2, 1} Mean: 2.416 Median: 2 Mode: 2

2. By comparing modes, which person went to the movies the least per month?

3. By comparing medians, which person went to the movies the most per month?

4. Rank the friends in order of most movies seen to least movies seen by comparing their means.

Appendix 6

Lesson Plan

Date: Day 2

Class: Statistics

Hour:

Learner Objectives: Compile data and use this data to find the mean, median, and mode for the game Chutes and Ladders.
Method: Play Chutes and Ladders in groups of two Worksheet
Content: Break into groups of two Give each group a game board and worksheet Go over rules as a class Have each group play Chutes and Ladders 20 times and keep track of their moves on the worksheet Have each group find the mean, median, and mode for the number of moves Have each group make a guess ~~and~~ as to the minimum and maximum game lengths Have each group prepare short presentation on their information for the next day

Appendix 6

Name_____

Chutes and Ladders

Here's the game plan...

- You and your partner play **TWENTY** games and keep track of your moves (you may want to use scratch paper for this).
- After each game, write down the number of moves of the person who **WON**.
- After twenty games, make a list and then find the mean, median, and mode and make a guess at the minimum and maximum game lengths.
- Also, be prepared to give a short (2-3 minutes) presentation on your information in class tomorrow!

Remember - A move is ONLY when your piece moves. For example, if you are on space 35 and roll a 6, you don't move your piece...DON'T count this as a move!

DATA ENTERED HERE!!

Game	Number of Moves	Game	Number of Moves
One		Eleven	
Two		Twelve	
Three		Thirteen	
Four		Fourteen	
Five		Fifteen	
Six		Sixteen	
Seven		Seventeen	
Eight		Eighteen	
Nine		Nineteen	
Ten		Twenty	

Make a list of the Number of Moves:

Mean:_____ Minimum:_____

Median:_____ Maximum:_____

Mode:_____

Appendix 6

Lesson Plan

Date: Day 3

Class: Statistics

Hour:

Learner Objectives:

Understand that a sample is not always perfect, but sometimes still useful to us
Also, an imperfect sample can yield useful information.

Method:

Board Work
Class Discussion
Overhead

Content:

Have groups come to board and present their data from the day before (about 2-3 min each)
Talk about the differences/similarities between each group's data
Why is this?
Compile all the classes info on an overhead
Find the mean, median, and mode of the compiled class data
Again, differences/similarities - Why?
Talk about minimum and maximum game lengths
Show moves of minimum game

Appendix 6

Overhead for Collecting Class Data

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Appendix 6

Overhead for Finding Mean, Median, and Mode of Class Data

Total Games:

Mean:

Median:

Mode:

Minimum:

Maximum: