

# **Using Maxwell's Z-Model to Determine Lunar Mare Lava Thickness**

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## Abstract

This project uses existing mathematical models of impact ejecta motion during crater formation to relate original material depth within a target to its final ejected distance from the crater. The resulting model equations were then used to estimate the maximum thickness of mare basalts at the locations of several lunar craters, and the results were compared Thomson's current work in the examination of lunar mare thickness.

## Introduction



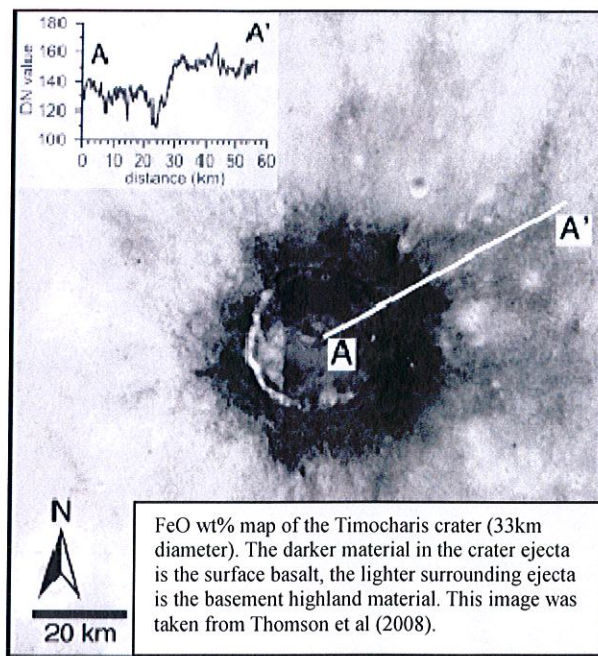
Remote sensing of impact craters may be one of the most effective ways to study planetary surface histories through depositional and erosional features. We can use this information to determine surface age, thermal evolution and surface rock origins. It is also a prime way to determine sub-surface material composition as well as type and effect of impactors. When a projectile strikes a planetary surface the material of the impactor itself and the material near the surface are ejected in a very fast, high-energy process. Though some of the material is vaporized on impact, much of it is distributed around the crater with the deepest material lying closest to the crater itself and the shallower material coming to rest far out into the rays of the ejecta (Shoemaker, 1963). A potential application of impact crater ejecta studies is in examining material under the surface that we cannot currently excavate.



The iron-rich lunar maria cover much of the Moon's surface; they are volcanic in origin and overlie older lunar highland crust which contains much less iron. When there is a small impact, ejecta contains only shallow mare material; when there is a larger impact, the ejecta will contain both iron-

rich mare basalt and also the deeper, low-iron basement rock. Material ejected on the surface will represent the inverted order of the layers below (Shoemaker, 1963). The difference in iron content is apparent under processing in the spectral imagery obtained by the Clementine spacecraft (See figure 1). When a crater is deep enough to have a significant amount of basement material in the ejecta, it appears like a halo around the crater at a roughly uniform radius. This radius can be used to determine the original material depth.

Figure 1.



Thomson *et.al* (2008) is currently using crater ejecta data from the lunar surface to examine the original thickness of the mare. His process estimates the basalt thickness using crater topographic and spectral data obtained from the Clementine Mission. An analysis of the iron content in the ejecta displays the radius of the basement material. The depth of the transient crater was inferred from known ratios of crater geometry. Thomson assumes that the ratio of continuous ejecta to that of the basement ejecta is linearly related to the corresponding material depths. From this assumption he developed a geometric relation to find basalt depth.



## History

The process that moves material during the formation of an impact crater is complex, but years of study have gone into the simplification of models that represent the motion with reasonable accuracy. Donald Maxwell (1977) developed a simple model for explosive cratering that has later been altered to work for impacts. The model describes the path that a particle follows from impact center along what is referred to as a streamline. The functions are based on the assumption that the flow is incompressible and that the trajectories of ejecta above the ground <sup>plane?</sup> plain are independent as a result of spall. Finally the model assumes that the radial component of velocity below the ground plain is derived from the strength flow coefficient over the polar radius to the power of Z. This Z in Maxwell's Z-model controls the shape or curve of the stationary stream lines below the surface.

In 1980, Steven Croft published a paper analyzing the Z model in terms of impact craters. His research included measurements of explosion craters where the charge was set at a depth below the surface, or at the EDOZ. He compiled information about ratios from radius to depth and he derived, from the Z model, a series of equations for most geologic features and geometric relations of a crater. He then applied these ratios to existing models of lunar maria depths and found that they were likely off by factors of two or three. He was also able to draw conclusions about the geometric similarities for craters of any size. From many of these assumptions, ratios can be drawn that are useful in later assumptions of this project.

The volumes of material ejected from lunar basins were examined by J.W. Head et al. in 1975.

He noted that the two ways to evaluate these volumes would be either to measure evidence of material around the crater and missing from within the crater basin, or to mathematically derive it with knowledge of small impact morphology. He noted that one third to one half of the apparent depth of a crater was actually created by compression in some materials, so the visual depth of a crater does not necessarily infer the depth of the material found in the ejecta. This is why finding depths through mathematical models (which assume incompressible flow) is essential for small impacts. Models are also more valid than using an assessment of surrounding ejecta for volumes because material in the ejecta may vaporize and may also contain impactor material.

Work done on flow-field center migration (Anderson et al., 2006) not only examined the theory that the depth of burst must be below the surface but actually tracked the movement of this depth as is related to time. Using NASA Ames Vertical Gun Range to simulate impacts, Anderson was able to observe the momentum transfer into the material and recognized that non-dimensional scaling for impacts only works when the impact angle is vertical. Anderson also noted that a consequence of applying Maxwell's model center at a depth below the surface results in having the earliest streamlines pointing back into the center of the crater making the angles higher than is represented in lab experimentation (Anderson et al. 2003).

Thomsen and Austin *et al.* did research using a small (0.3g) impactor into plasticene clay and examined the effects on Maxwell's model of a time dependent flow strength coefficient ( $\alpha$ ) and Z. Data for the lab experiments were recorded in early-time (through 18 $\mu$ sec) and also in later-time (18 $\mu$ sec to 600 $\mu$ sec) growth of the crater. Regressions were applied to account for the variability of  $\alpha$  and Z dependent on different depths of the flow field center. This experiment was also expanded to using gabbroic anorthosite (similar to lunar composition) as the impact material. It was from these



works that our assumptions about alpha were derived (Thomsen et al. 1979, Austin et al. 1980, Austin et al. 1981).

## Method

For this study, we use Maxwell's Z model, incorporating modifications by Croft, to derive estimates of maximum mare lava thickness at several locations. Several assumptions are required to develop mare thickness estimates. These assumptions and the equations used will be expanded on now.

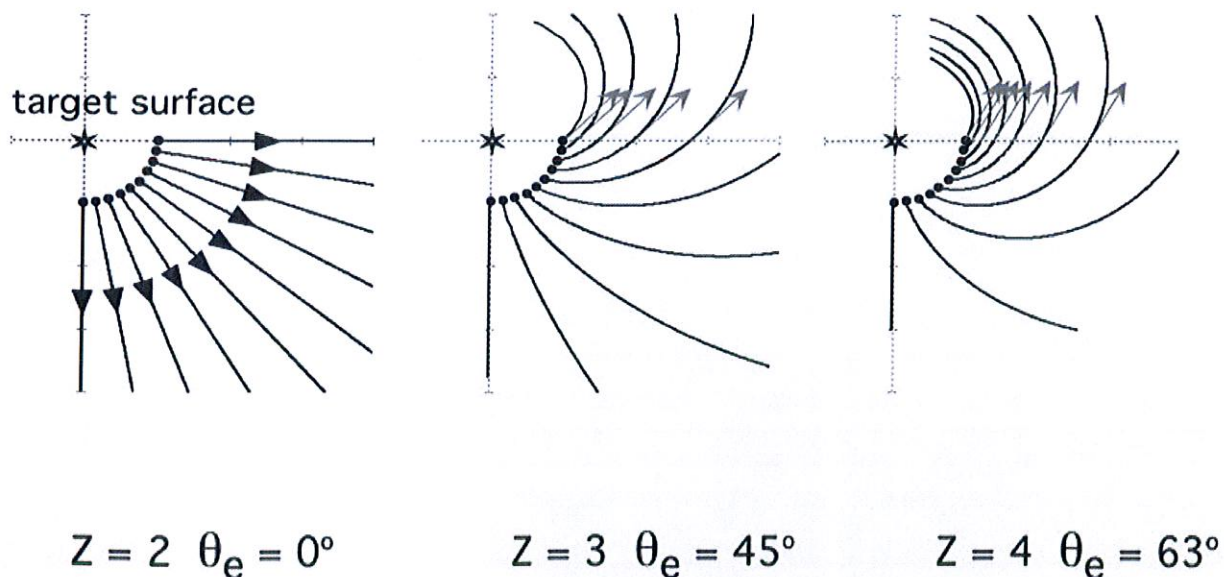
### Initial Assumptions

The first assumption for this project is that craters were formed from a near vertical impact. While it has been shown that most impacts happen at 45 degree angles or less from the surface (Shoemaker, 1962); a vertical impact model represents all but the lowest angle impacts with a reasonable degree of accuracy for the extent of this project (Anderson et al. 2006).

Using a lunar environment established many other assumptions for this project. This environment is ideal because of the simplicity of ejecta motion in a vacuum. A lunar environment is also highly relevant to the outcome of understanding material depth to ejecta distance relations because of the layer of Maria over much of the lunar surface. Assuming a lunar environment establishes gravity to be about  $1.622 \text{ m/s}^2$  and for convenience, also establishes the units for the rest of this project to be in meters. It also establishes a lack of atmospheric drag.

Because the data for this project were derived from known relations rather than lab experimentation, several assumptions had to be made about the constants, ratios, and other aspects of the impact. Maxwell's Z-model, which is a point source model used for its simplicity and analytical development, demonstrates a correlation to existing data to a good degree for this project. Three of the most important parameters in this model are the Z which characterizes the streamline shape (figure 2), the strength flow coefficient alpha, and the effective depth of the Z-model (EDOZ). Austin and Thomsen, *et al.* worked with .3g projectiles in plasticene modelling clay and evaluated Maxwell's model for alpha and Z during the early formation of the crater. Their results found Maxwell's model reasonable assuming the Z-model center was below the surface by about one projectile diameter (Thomsen et al. 1979). This is confirmed by Croft (1980). They also continued the study into the later-time crater growth and examined the effect of a constant or variable Z and alpha. It was found that if alpha and Z are held fixed, the EDOZ must move downward to maintain agreement with their data. In other words, both alpha and Z displayed a time dependant variable relationship throughout crater growth (Austin et al. 1980).

Figure 2: Z and Streamline Shape.



This figure was taken directly from Anderson *et al.* (2004) and uses Croft's (1980) modified streamline equation from Maxwell's Z model.



In the work done by Croft (1980) there is an agreement that a constant  $\alpha$ ,  $Z$  and EDOZ cannot accurately represent an impact. However he also found that the  $Z$  and EDOZ can remain constant with a variable  $\alpha$  and still represent the streamlines to a good degree, especially during the latter half of the crater excavation, which is the portion of interest for this study. In this study, we chose to follow this idea and assumed a constant  $Z$  and EDOZ and developed an equation for a time-dependant  $\alpha$ . The  $Z$  for this study was kept at 2.71 which was found reasonable in Maxwell's work and was used in other research including Croft (1980), Austin *et al.* (1981), and Anderson *et al.* (2003). We also used an EDOZ of about one projectile diameter (Austin et al 1980, Croft 1980).

The projectile diameter itself had to be derived from the final size of the crater; this was done assuming, as we can with many other aspects of crater geometry, that there is a relatively constant ratio in sizes. In this case data was used from Austin *et al.* (1981) in which a calculation was run for an iron projectile of 62.4 meters in diameter, projected into a gabbroic anorthosite material (which is similar to the lunar surface composition) leaving a 900 meter diameter crater. This ratio means the impactor radius is then roughly 0.138 the size of the final crater radius. This ratio was used for the EDOZ.

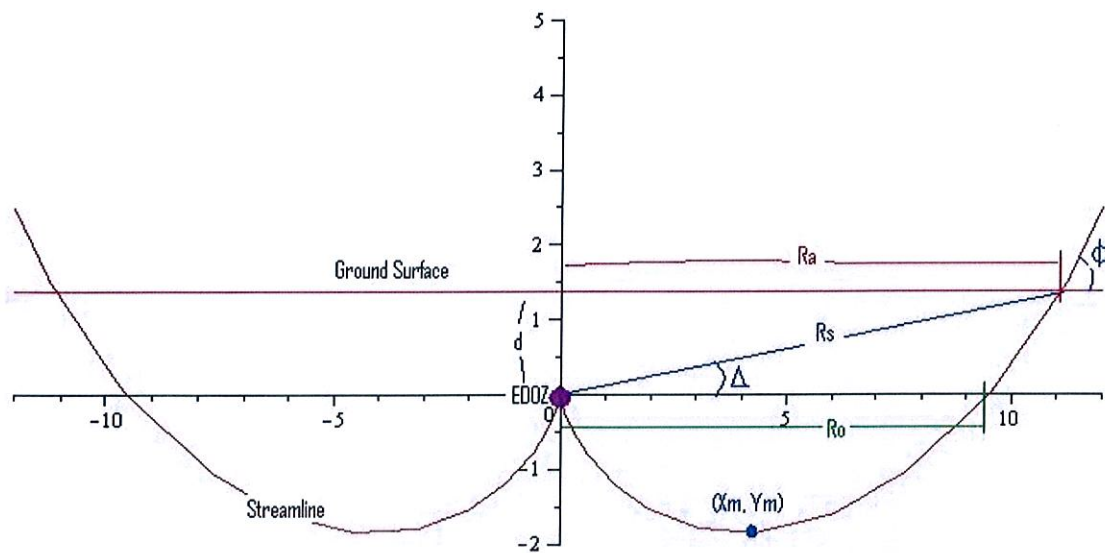
### Mathematical Assumptions

Many of the following assumptions were derived from the appendix of Croft's work (1980) and will use the same notation (See fig. 3). A table was built through Microsoft Excel using these formulas to show the geometric relations as a function of the percent of the crater growth. All relations are



initially dependant on the final apparent radius size which can be entered into a single box in meters to recalculate the contents of the table. Each column of the table is a different variable in the crater geometry and the derivations for these variables will now be explained.

Fig 3. Geometry of a Streamline.



This diagram shows a streamline originating at the axis center at depth  $d$  from ground level.  $R_o$  is the radius of the crater parallel with the EDOZ.  $R_a$  is the radius from the center of impact to the edge of the crater rim at the surface, and  $R_s$  is the radius from the EDOZ to the surface crater rim.  $\Delta$  is the angle from horizontal from the EDOZ to the ground surface at the final crater rim and  $\phi$  is the angle of the ejecta trajectory at the surface. The point  $(X_m, Y_m)$  is the horizontal and vertical coordinates of the maximum depth of the streamline.

The apparent radius or  $R_a$  of a crater is the final visual radius at the surface. Throughout formation of the crater this radius is only a percentage of its final size and so this percentage was used as a marking point for various stages throughout crater growth. Below this surface level is the Z-model origin at the EDOZ noted previously which is at depth  $d$ . From this EDOZ to the far edge of the  $R_a$  is a length denoted as  $R_s$  which is the Z-model radius at the surface level. It was calculated by:

$$1) R_s = \sqrt{(d^2 + R_a^2)}$$

The angle from the horizontal from this EDOZ center to the edge of  $R_a$  along  $R_s$  is noted by  $\Delta$ .

$$2) \Delta = \arctan (d/R_a)$$

A simplified streamline equation is given by Croft (1980)

$$3) R = R_o(1 - \cos(\theta))^{\frac{1}{Z-2}}$$

Knowing  $R_s$ , which is the radius of the streamline equation at the surface, it becomes possible to solve for  $R_o$ , which is the radius of the streamline equation parallel with the EDOZ.

$$4) R_o = R_s / ((1 + \sin(\Delta))^{1/(Z-2)})$$

$Y_m$  is the maximum depth of the final crater and is given by Croft (1980) as:

$$5) Y_m = R_o(Z-2)[Z-1]^{\frac{1-Z}{Z-2}} + d.$$

The final points of detail are relating to the trajectory and material in the ejecta above the ground surface. The first of these functions is the velocity and angle of ejection which is dependent on  $\alpha$  and was taken from Maxwell (1977). The radial and angular components are:

$$6) U_r = \alpha R^{-Z}$$

$$7) U_\theta = \alpha R^{-Z} * (Z-2) * \sin \theta / (1 + \cos \theta)$$

In these equations  $R$  is at the surface so  $R_s$  can be used. From these equations  $U_x$  and  $U_y$  can be derived, the derivations presented by Croft (1980) were used. Also taken from Croft's appendix was the ejection angle  $\phi$ :



$$8) \phi = \tan^{-1} * \left[ \frac{\frac{\tan \Delta}{\cos \Delta} - \tan^2 \Delta + (Z-2)}{\frac{1}{\cos \Delta} - \tan \Delta (Z-1)} \right]$$

The distance from the crater rim to the final location of the ejecta can be derived from trajectory equations knowing the gravity and horizontal velocity of this distance or  $X_g$  is:

$$9) X_g = V_x * 2V_y/g + R_a.$$

### The Flow Strength Problem

When developing the relations for impact craters in lunar conditions, it was noted that a constant strength flow coefficient alpha gave unrealistic results, which was consistent with Croft's (1980) conclusions that  $Z$ , EDOZ, and alpha cannot all be kept constant. But having a time-dependent alpha requires a function to relate it to the crater formation. This was developed using the data collected by Austin *et al.* (1980) for a .3 g aluminum projectile in plasticene clay material during crater development from 18  $\mu$ sec to 600  $\mu$ sec.

Austin's published results for alpha at 90 degrees from EDOZ were plotted against time. Also published were the crater depths throughout growth during the same time frame. Using those depths and Croft's derivation for the maximum depth, (Croft 1980) the radius  $R_o$  was derived for those times. Having both alpha and crater radius as time dependant functions allowed the two functions to be plotted against each other, and the crater radius was converted into a percent of the final crater size to negate the units. When this data was plotted, Microsoft Excel was used to find a suitable power regression. This regression showed the flow strength to be:

$$10) 4014.1 * R_a^{-1.861}.$$

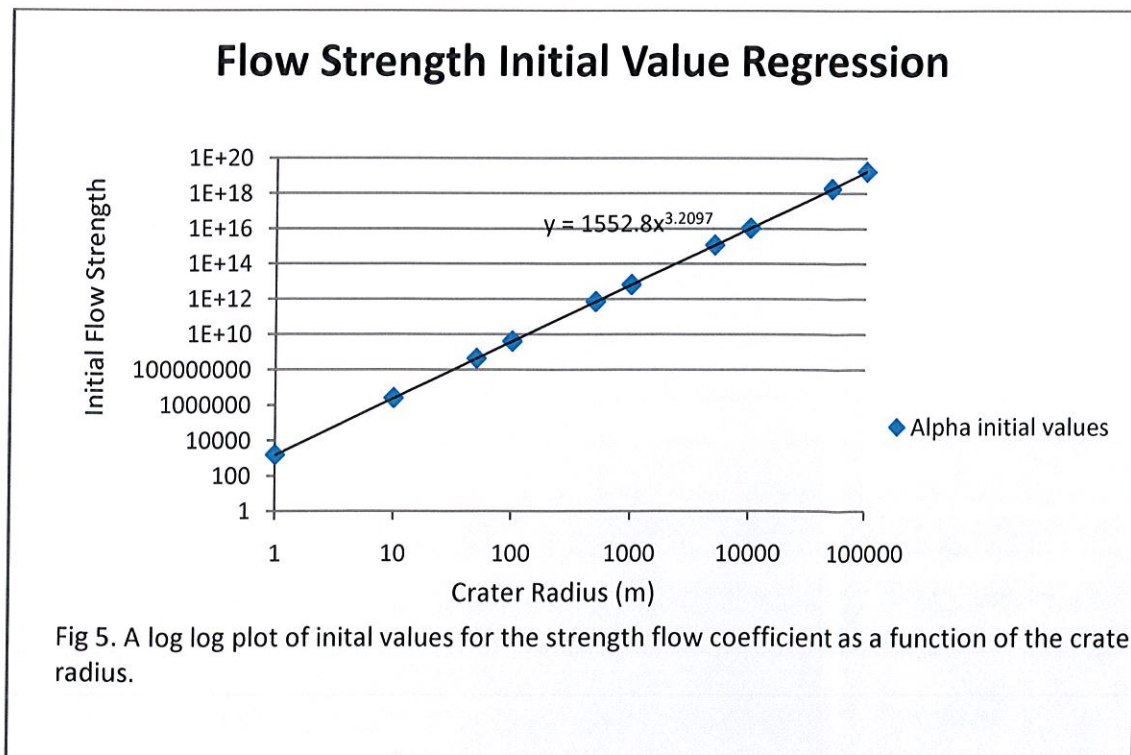
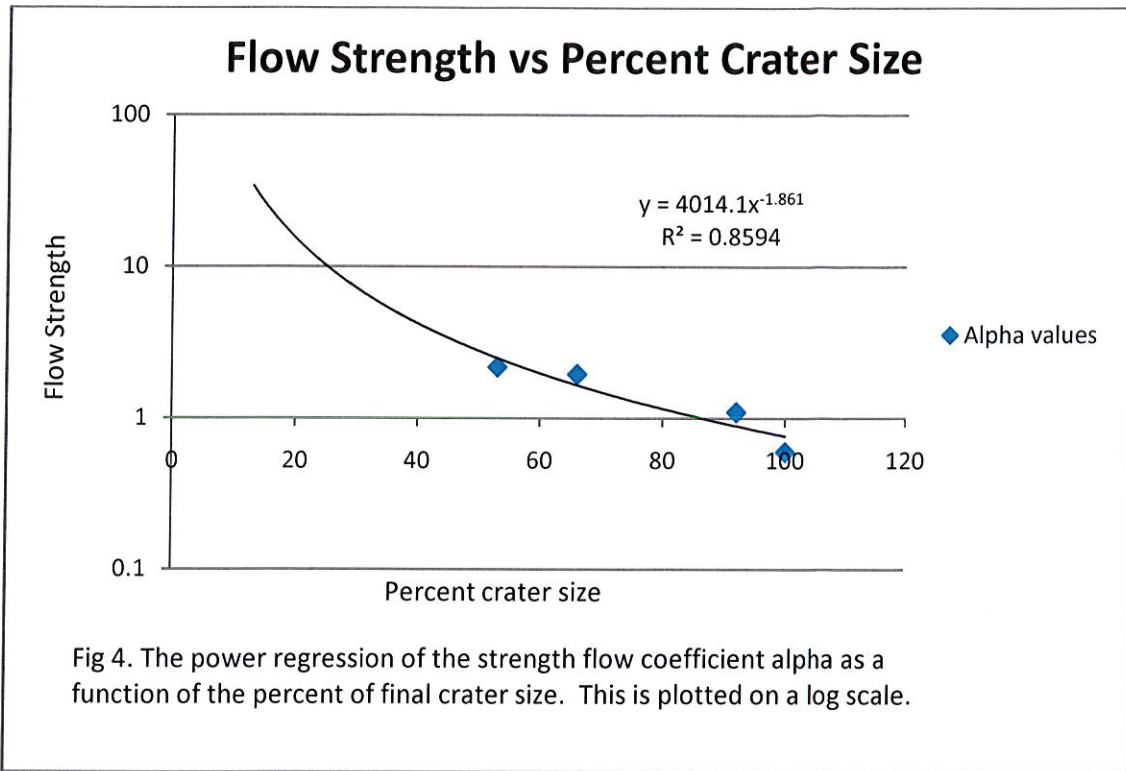
During the latter half of crater formation, the flow strength alpha declines by  $\sim R^{-1.78}$ . This is shown in

Fig 4, plotted with the y-axis on a log scale.

After this regression was applied to the data, it was discovered that the initial number in the function is dependent on the size of the crater. In other words, alpha gave realistic results for a calculated crater of one meter radius, but the function did not apply for larger or smaller craters. Another regression was developed so that the alpha function could be applied to any size crater. In doing this we recognized the initial number (4014.1) actually had to be dependent on crater size. We called this initial number  $\alpha_0$ .

To find  $\alpha_0$  we recognized that in the last stages of ejecta flow, the final material trajectories should be nearly the same, leaving the distance for the final ejecta locations to be similar. This is a result of the overturned flap, as noted by Maxwell (1977) referring to the low energy trajectories of the final streamlines. Numbers were experimentally placed in the position of  $\alpha_0$  until the same value for the final two ejecta distances was attained. That number was plotted against the crater radius for which it was found. When several data points were established, a power regression was fit to the points plotted in a log-log scale. The result of the power regression is in Fig 5. In order to produce the required order of ejecta deposition,  $\alpha_0$  must increase as  $\sim Ra^{3.2}$ .





## Results

The data derived from this process has been formatted into a Microsoft Excel table that is dependent on the number entered in for apparent crater radius in meters. The table shows maximum depths for streamlines throughout crater growth as well as other geometric aspects of the crater morphology. This table has been included in an appendix for a crater of 10 km. Though streamline volumes were calculated for this project, they were not used in the final relations and so they have not been included in this table.

The maximum depths for the basalt were inferred assuming that any ejecta containing the low iron basement material could be detected in the Clementine imagery. The crater size was entered into the data table and the radius of basement material was related to the corresponding maximum streamline depth for that streamline. This only gives a maximum mare depth and is expected to be a high estimate because it is likely that basement material would not be detected at such a small percent of overall streamline volume.

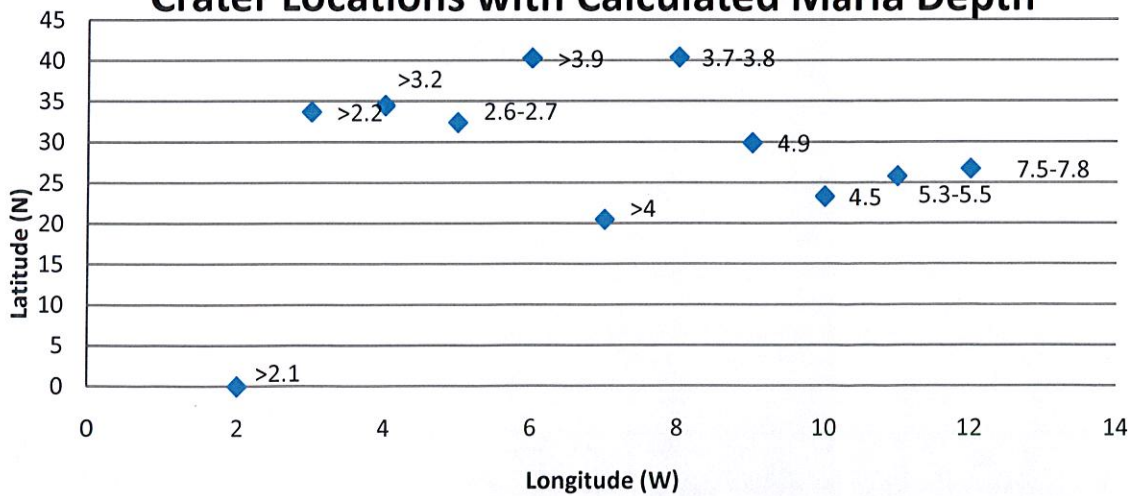


**Table 1. Locations and geometry of particular lunar craters and estimated basalt depths.**

Crater	Lat (N)	Lon (W)	D (km)	radius b (km)	Thomson depth (km)	My estimate (km)
Carlini	33.7	24.1	10	n.a	>.84	>1.6
C.						
Herschel	34.5	31.2	13	n.a	>1.09	>2.1
Heis	32.4	31.9	14	n.a	>1.18	>2.2
Le Verrier	40.3	20.6	20	n.a	>1.55	>3.2
Pytheas	20.5	20.6	20	19.9	0.44	2.6-2.7
Helicon	40.4	23.1	24	n.a	>1.81	>3.9
Delisle	29.9	34.6	25	n.a	>1.9	>4
Euler	23.3	29.2	27	22.5	1.04	3.7-3.8
Lambert	25.8	21	30	n.a	>2.0	4.9
Timocharis	26.7	13.1	33	30.6	0.95	4.5
Autolycus	30.7	-1.5	39	35	1.17	5.3-5.5
Aristillus	33.9	-1.2	55	49.4	1.58	7.5-7.8

The crater data (location, Crater diameter (D) and radius of basement material (radius b) in the ejecta) were derived from Clementine images formatted by Thomson et al. in his current work. His inferred depths and my estimates for the maximum depths are shown. N.a in the radius column implies the lower layer wasn't reached.

### Crater Locations with Calculated Maria Depth



This table shows the locations of craters used in this study and my results for maximum basalt depth in those locations

## Discussion

The data obtained for the geometric relations for a crater are reasonable and expected for the given conditions. The numbers are also roughly comparable to those found by Thomson et al. (2008). Our calculations show the basement ejecta to be deeper than Thomson's by a factor of about four. This is potentially the result of assuming that any basement material in the streamline will be apparent in the ejecta. It would be more realistic, however, to assume that the basement material will only be apparent as it approaches about 50 percent of the streamline volume. The data also shows a very clear trend toward a deeper calculated basement material for a larger crater. This flaw in the data is still unknown in origin but its consistency implies that it is derived from a single error.

There are a few issues of note with the assumptions made for this project. One of the major assumptions made was that craters shared the same morphology at nearly any size. It has however, been noted that the morphology for craters larger than 15 km may not in fact be accurately modeled by Maxwell's formulas (Head, 1975). This is also similar to the flawed assumption that all impacts are vertical when in fact they most likely are not (Shoemaker, 1962).

Initially it was assumed that the strength flow coefficient would be constant, however this resulted in inverted ejecta trajectories. A value for the strength flow ultimately based as a function of time is much more accurate; however the basis for our derivation of alpha may be flawed. The lab impacts used from Austin and Thomsen's research (1979, 1980) were based in plasticene clay, which has no particular shared similarity with lunar soil. In fact, a similar project done in gabbroic anorthosite material, which is similar to lunar material, yields very different results for the flow strength in terms of time (Austin 1981). We could not use this data, however, because of the lack of comparable



basement material as a percentage of the total streamline volume. The basement material should only be considered to be apparent when it approaches 50 percent of the total ejecta volume.

To develop the relations of basement material volume, one must first determine the volume of the entire streamline. This is done by Croft (1980) in his appendix.

$$11) V_{sl} = 2\pi \int_0^{R_o} \int_0^{90+\Delta} r^2 \sin\theta \, dr \, d\theta + \frac{\pi}{3} d R a^2$$

This is the volume from the streamline to the surface, and the volume of only the streamline can be deduced by subtracting the volume of the previous streamline from the next. To determine the total volume of the basement material within the streamline the equation for the streamline would be converted into two functions, one for each side of the maximum depth. Setting  $f(x)$  to be the function where  $x$  is greater than the maximum, and  $g(x)$  to be the function where  $x$  is less than the maximum, the volume could be determined as follows:

$$12) V_b = \pi \int_k (f(x)^2 - g(x)^2) dx$$

where  $k$  is the depth of the basement material. This volume would also have to be broken into streamlines and then percentages of the total streamlines derived.

To continue this project work would also be done to discover the cause for the trend in basalt depth correlated with crater size. This is an obvious flaw, but there is not time to ascertain the origins. It is suspected though, that it may actually be correlated with our alpha derivation. Along these lines, it would also be worthwhile to assess the accuracy of Thomson's assumption that material depth ratios are linearly correlated to ejecta material radius ratios.

## Conclusions

Maxwell's model and derivations were used to relate original material depth to a final ejected distance from an impact crater. Data developed from these derivations was then applied to an estimation of maximum basalt thickness of lunar mare. These results were directly compared to similar estimates developed by Thomson (2008).

There were some interesting results from the process of deriving alpha that were mentioned in the discussion. These results include a correlation between the strength flow coefficient and the peak shock wave stress decay, which is expected but has not previously been noted in other work. The study of alpha also lead to some potential insight about the debate on final crater size scales from initial momentum and kinetic energy of the impactor.

It can be assumed that the process used to derive these basalt estimates is relatively accurate due to the similarity of the data to existing estimates. There is an obvious flaw in the data resulting in a deeper basalt depth estimate for a larger crater size that still needs to be addressed. Also it is important to establish that this method currently only derives the maximum possible depths of the basalt.

It would be worthwhile to recalculate these depths with a more accurate assessment based on a higher basement material content required for detection in the imagery. This would also bring the numbers into closer agreement with other conclusions on lunar basalt depth. These calculations would be made using volumes of the streamlines and volumes of material in the streamline below the mare.

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## Appendix

**Table 1: Data for a 10,000 meter crater**

Key		%final Ra	Ra	Rs	Δ (deg)	Ro	Ym	α	Ur	Uθ	Ux	Uy	φ (de)	X(G)
Z	2.71	0.2	200	2429.8	34.6	1674.	1706.	3.983E	26624.	36010.	1463.	44759.	88.1	80752916
G	1.62		0	97	06	930	660	+13	587	112	115	983	28	.829
(m/s^2)	2		250	2855.5	28.8	2101.	1789.	2.63E+	11348.	13651.	3338.	17435.	79.1	71771151
		0.25	0	91	99	125	780	13	549	259	236	671	61	.569
			300	3302.1	24.7	2554.	1878.	1.873E	5452.1	6041.6	2428.	7767.2	72.6	23260873
		0.3	0	81	02	637	228	+13	75	44	412	70	38	.459
			350	3762.2	21.5	3025.	1970.	1.406E	2873.9	2997.8	1573.	3843.1	67.7	7462066.
		0.35	0	33	19	483	056	+13	18	95	963	00	28	207
			400	4231.3	19.0	3507.	2064.	1.097E	1630.2	1623.7	1011.	2066.6	63.9	2581641.
		0.4	0	59	34	513	066	+13	29	35	534	31	20	372
			450	4706.8	17.0	3996.	2159.	8.807E	981.10	942.22	661.7	1188.4	60.8	974237.3
		0.45	0	46	49	895	510	+12	6	4	40	68	91	55
			500	5186.9	15.4	4491.	2255.	7.239E	619.79	577.96	443.6	722.03	58.4	400015.7
		0.5	0	45	30	190	911	+12	6	4	89	2	29	61
			550	5670.4	14.0	4988.	2352.	6.062E	407.67	371.04	305.1	459.10	56.3	178229.1
		0.55	0	85	85	816	963	+12	8	9	20	8	92	04
Radius	100		600	6156.6	12.9	5488.	2450.	5.156E	277.44	247.43	214.9	303.33	54.6	86385.92
(m)	00	0.6	0	55	53	725	459	+12	6	7	24	0	81	3
EDOZ	138		650	6644.8	11.9	5990.	2548.	4.442E	194.38	170.39	154.7	207.05	53.2	46011.32
(m)	0	0.65	0	78	86	208	263	+12	9	4	63	0	23	0
	1E+		700	7134.7	11.1	6492.	2646.	3.87E+	139.65	120.61	113.6	145.34	51.9	27375.63
α0	16	0.7	0	32	52	779	279	12	7	3	91	8	68	7
			750	7625.9	10.4	6996.	2744.	3.404E	102.55		85.03	104.54	50.8	18462.15
		0.75	0	03	26	100	441	+12	2	87.432	7	6	75	9
			800	8118.1	9.78	7499.	2842.	3.019E			64.64		49.9	14123.39
		0.8	0	52	7	931	702	+12	76.765	64.710	8	76.817	17	3
			850	8611.2	9.22	8004.	2941.	2.696E			49.87		49.0	12036.92
		0.85	0	95	2	101	030	+12	58.447	48.778	4	57.513	69	3
			900	9105.1	8.71	8508.	3039.	2.424E			38.99		48.3	11105.14
		0.9	0	85	7	488	400	+12	45.178	37.369	2	43.785	14	0
			950	9599.7	8.26	9013.	3137.	2.192E			30.85		47.6	10787.23
		0.95	0	08	5	003	795	+12	35.398	29.047	5	33.834	37	4
			100	10094.	7.85	9517.	3236.	1.993E			24.68		47.0	10806.55
		1	00	771	7	581	202	+12	28.076	22.874	6	26.498	27	1

This table shows the data for a 10 km crater, the assumptions for variables are noted previously.